**Time: 3 Hours**            **Marks: 80**

**N.B:**    (1) Question no 1 is compulsory

         (2) Attempt any **three** out of remaining **five** questions

         (3) Figures to the right indicate full marks

         (4) Assume Suitable data if necessary

         (5) Notations carry usual meaning

Q.1    Answer **any four** of the following:

a)For the game $\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$ where a,b,c,d are all $\geq 0$ prove that the optimal strategies are:

A$[\frac{c+d}{a+b+c+d}, \frac{a+b}{a+b+c+d}]$; B $[\frac{b+d}{a+b+c+d}, \frac{a+c}{a+b+c+d}]$ and v$=\frac{ad-bc}{a+b+c+d}$.    **(05)**

b)Solve the LPP by Graphical method:

Maximize Z $=100x_1 + 80x_2$

Subject to $5x_1 + 10x_2 \leq 50$

$8x_1 + 2x_2 \geq 16$

$3x_1 - 2x_2 \geq 6$

And $x_1 \ and \ x_2 \geq 0$    **(05)**

c)Explain Bellman's Principle of optimality    **(05)**

d) How do you test initial feasible solution for optimality by Stepping Stone method.**(05)**

e)Define a queue. State the characteristics of waiting line.    **(05)**

Q.2    a) A jobbing shop has three machine groups, namely lathes, milling machines and grinders. It has an idle capacity of 350 hours, 500 hours and 150 hours per week respectively. It is offered products A, B and C to be manufactured. Each unit of product A yields RS.30,product B Rs.12 and product C Rs.15.The time taken by each unit of the three products on different machines is given in Table below:

| | Lathe | Milling | Grinder |
|---|---|---|---|
| **Product A** | 5 | 9 | 3 |
| **Product B** | 4 | 3 | 0 |
| **Product C** | 0 | 5 | 2 |

     i)      How much quantity of products A, B and C must be manufactured every week to yield maximum profit?

     ii)      What capacity of each machine remains idle after making these products? State clearly the assumptions if any, made by you.    **(10 marks)**

b) A dentist schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on type of dental work to be done. The following summary shows the various categories of work, their probabilities and time needed to complete the work

| Category | Time required (minutes) | Probability of category |
|---|---|---|
| Filling | 45 | 0.40 |
| Crown | 60 | 0.15 |
| Cleaning | 15 | 0.15 |
| Extraction | 45 | 0.10 |
| Check-up | 15 | 0.20 |

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic exactly as per scheduled arrival times, starting 8 A.M.Use following random numbers for handling the above problem.40,82,11,34,25,66,17 and 79. **(10 marks)**

Q.3  a) A captain of a cricket team has to allot five middle order batting positions to five batsmen. The average runs scored by each batsman at these positions are given in the table.

| Batting Position / Batsman | III | IV | V | VI | VII |
|---|---|---|---|---|---|
| A | 40 | 40 | 35 | 25 | 50 |
| B | 42 | 30 | 16 | 25 | 27 |
| C | 50 | 48 | 40 | 60 | 50 |
| D | 20 | 19 | 20 | 18 | 25 |
| E | 58 | 60 | 59 | 55 | 53 |

Make the assignment so that the expected total average runs scored by these batsmen are maximum. **(10 marks)**

b) Customers arrive at one person barber shop according to Poisson process with mean inter arrival time of 20 minutes. Customers spend on an average of 15 minutes in the barber's chair.
 i) What is the probability that a new arrival need not wait for the barber to be free?
ii) What is the expected number of customers in the barber shop?
iii) How much time can a customer expect to wait for his turn?
iv) How much time can a customer expect to spend in the shop?
v) Management will put in another and hire another barber when customer's average time in the shop exceeds 1.25 hours. How much must the average rate of arrivals increase to warrant a second barber? **(10 marks)**

Q.4  a) The following table has all necessary information on availability of supply to each warehouse, the requirement of each market and unit transportation cost (in Rs.) from each warehouse to each market.

| | | Market | | | | |
|---|---|---|---|---|---|---|
| | | P | Q | R | S | Supply |
| Warehouse | A | 6 | 3 | 5 | 4 | 22 |
| | B | 5 | 9 | 2 | 7 | 15 |
| | C | 5 | 7 | 8 | 6 | 8 |
| Demand | | 7 | 12 | 17 | 9 | |

The shipping clerk has worked out the following schedule from his experience:
12 units from A to Q,1 unit from A to R,8 units from A to S,15 units from B to R,7 units from C to P,1 unit from C to R.
   i)      Check and see if the clerk has worked out the optimal schedule
   ii)     Find optimal schedule and minimum total transportation cost.     **(10 marks)**

b) There are two players in a game, Player A and Player B.Each of them randomly shows selected fingers of his right hand. If the sum of the number of fingers shown by both the players is an even number, then player B has to give money in rupees equivalent to the number of fingers shown by him to Player A; if the sum of number of fingers shown by both the players is an odd number, then Player A has to give money in rupees equivalent to the number of fingers shown by him to Player B.Construct the playoff matrix with respect to Player A and find optimal solution for this game.     **(10 marks)**

Q.5   a) A manufacturer is offered two machines A and B.A is priced at Rs.10,000 and running costs are estimated as Rs.1600 for each of the first five years, increasing by Rs.400 per year in the sixth and subsequent years. Machine B which has the same capacity as A, costs Rs 5000 but will have running costs of Rs.2400 per year for six years, increasing by Rs.400 per year thereafter. If money is worth 10% per year, which machine should be purchased? Assume scrap value to be negligible.     **(10 marks)**

b) The demand for an item is deterministic and constant over time and it is equal to 24000 units per year and company can produce the item at a rate of 4000 per month. The cost of one setup is Rs.600 and the carrying cost is 20 paisa per unit per month. The shortage cost of one unit is Rs.300 per year. Determine:
i) The optimum manufacturing quantity and no. of shortages
ii) The manufacturing time and the time between setup
iii) Maximum inventory level in the cycle and
iv) Total associated cost per year if the cost of an item is Rs.25 per unit.     **(10 marks)**

Q.6   a) Using **Big-M** method,
Minimize Z=3x+8y
Subject to $x + y = 200$
          $x \leq 80$
          $y \geq 60$
Where $x, y \geq 0$     **(10 marks)**

b) Given the following data:

| Job | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Machine A** | 12 | 10 | 9 | 14 | 7 | 9 |
| **Machine B** | 7 | 6 | 6 | 5 | 4 | 4 |
| **Machine C** | 6 | 5 | 6 | 4 | 2 | 4 |

   i.     Order of processing: A-C-B
   ii.    Suggested sequence of Jobs: 5-3-6-2-1-4
   iii.   Determine the total elapsed time for sequence suggested.
   iv.    Is the given sequence optimal?
   v.     If No, then determine the optimal sequence and total elapsed time associated
          with it.     **(10 marks)**