DISCRETE STRUCTURE   **Q.P. Code: 24629**

Duration: 3hrs   **[Total Marks: 80]**

1) Question **no.1** is **compulsory**.
2) Solve any **three** questions out of remaining **five** questions.
3) All questions carry equal marks as indicated by figures to the right.
4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Prove by induction that n^2+n is an even number, for every natural number n.   **(05M)**

b) Find the generating function for the following finite sequences   **(05M)**
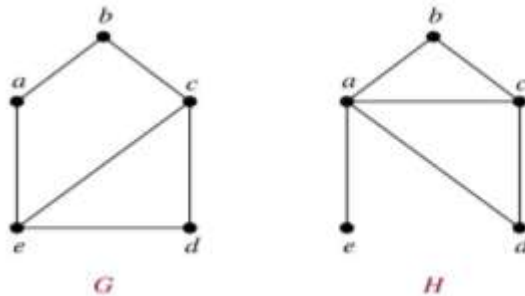
   i) 2,2,2,2,2,2   ii) 1,1,1,1,1,1

c) Let A={a,b,c,d,e} and R={(a,a),(a,b),(b,c),(c,e),(c,d),(d,e)}   **(05M)**

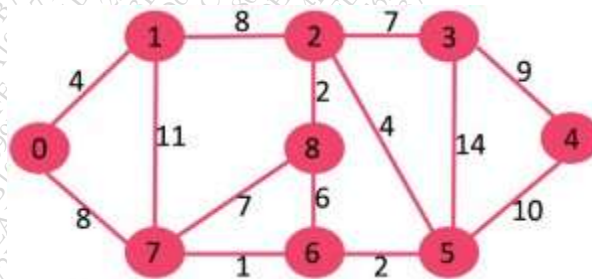   Compute R^2 and R^∞

d) Define Lattice. Check if the following diagram is a lattice or not.   **(05M)**

Q.2 a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the
one-to-one correspondence between the vertices. If not justify your answer.   **(08M)**

b) Find Minimum spanning tree for the following graph using Kruskal's Algorithm.   **(08M)**

c) Prove $(p \vee q) \wedge (p \wedge \neg q) \vee q] \leftrightarrow p \vee q$

Q. 3 a) Prove that set G = {0,1,2,3,4,5} is a finite abelian group of order 6 with respect to addition modulo 6. **(08M)**

b) Let A={1,2,3,4}, let R={(1,2),(2,3),(3,4),(2,1)} Find Transitive closure of R using Warshall's Algorithm. **(08M)**

c) Test whether the following function is one-to-one, onto or both. **(04M)**

f: $Z \to Z$, $f(x)=x^2+x+1$

Q.4 a) Show that the (2,6) encoding function $e:B^2 \to B^5$ defined by **(08M)**

e(00)=000000   e(01)=011110

e(10)=101010   e(11)=111000 is a group code.

Find Minimum distance.How many errors will it detect and correct?

b) Let H = $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H. 1) 01111 2)01110   3)11001 **(08M)**

c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? **(04M)**

Q.5 a) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a*b=a+b-ab for all a,b €G. Prove that (G,*) is a group.

b) Solve the recurrence relation $a_r-a_{r-1}-6a_{r-2}=-30$ given $a_0=20$, $a_1=-5$ **(08M)**

c) Let A={a,b,c,d,e,f,g,h}. Consider the following subsets of A **(04M)**
A1={a,b,c,d}   A2={a,c,e,g,h}
A3={a,c,e,g}   A4={b,d}   A5={f,h}
Determine whether following is partition of A or not. Justify your answer.
i) {A1, A2}   ii) {A3, A4, A5}

Q.6 a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers. **(08M)**

I.   A={2,4,,12,24}
II.   A={1,3,5,15,30}

b) Let the functions f,g, and h defined as follows: **(08M)**
f:$R \to R$, f(x)=2x+3
g: $R \to R$, g(x)=3x+4
h: $R \to R$, h(x)=4x
Find gof, fog, foh, , gofoh

c) Determine Euler Cycle and path in graph shown below (**04M**)