Q.P. Code : 37077

(3 Hours) **[Total marks : 80**

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) Find the Laplace transform of $sinh^5 t$. 05

b) Find an analytic function whose imaginary part is $e^{-x}(y\cos y - x\sin y)$. 05

c) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. 05

d) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = 2x\, i + (xz - y)\, j + 2z\, k$ from $O(0,0,0)$ to $P(3,1,2)$ along the line OP. 05

Q.2 a) Find a cosine series of period 2π to represent $\sin x$ in $0 \le x \le \pi$. 06

b) Find a, b, c if $\bar{F} = (axy + bz^3)\, i + (3x^2 - cz)\, j + (3xz^2 - y)\, k$ is irrotational. 06

c) Find the image of the circle $|z| = k$ where k is real under the bilinear transformation $w = \frac{5-4z}{4z-3}$. 08

Q. 3 a) Prove that $J_{\frac{1}{2}}(x) = \tan x \cdot J_{-\frac{1}{2}}(x)$. 06

b) Find the inverse Laplace transform of the following function by convolution theorem 06
$\frac{(s+2)^2}{(s^2+4s+8)^2}$.

c) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$ where a is not an integer. 08

Q. 4 a) Find the angle between the normals to the surface $xy = z^2$ at the points $(1, 4, 2)$ and $(-3, -3, 3)$. 06

b) Prove that 06
$x^2 J_n''(x) = (n^2 - n - x^2)\, J_n(x) + x\, J_{n+1}(x)$;
$n = 0, 1, 2, \ldots\ldots$

c)
(i) Find the Laplace transform of $sinhat \sin at$. 04

(ii) Find the Laplace transform of $te^{-4t} \sin 3t$. 04

Q. 5 a) Prove that $J_2(x) = J''_0(x) - \frac{J_0'(x)}{x}$. 06

b) If $v = e^x \sin y$, show that v is harmonic and find the corresponding analytic function. 06

c) Find the Fourier series for $f(x)$ in $(0, 2\pi)$, 08

$$f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi \le x < 2\pi \end{cases}$$

Hence, deduce that

$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$

Q. 6 a) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \ldots\ldots$ is orthogonal on $(0, 2\pi)$. 06

b) Using Green's theorem evaluate $\int_C \bar{F} \cdot d\bar{r}$ where C is the curve enclosing the region bounded by $y^2 = 4ax, x = a$ in the plane $z = 0$ and 06

$$\bar{F} = (2x^2y + 3z^2)\, i + (x^2 + 4yz)\, j + (2y^2 + 6xz)\, k.$$

c) Use Laplace transform to solve 08

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ with $y(0) = 0, y'(0) = 1.$

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