

Max Marks : 80

Time : 3 Hours

N.B. : 1) Question number 1 is **compulsory**

2) Answer **any three questions** from the remaining questions

3) Figures to the right indicate full marks

Q.1 a) Solve $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$ (5)

b) Evaluate $\int_0^1 \sqrt{x-x^2} dx$ (5)

c) Solve $(D^2 + 4)y = \cos(2x)$ (5)

d) $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2) dy dx$ (5)

Q.2 a) Assuming the validity of differentiation under the integral sign, Prove that (6)

$$\int_0^1 \frac{x^a - x^b}{\log x} dx = \log \frac{a+1}{b+1}$$

b) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \frac{e^{-2x}}{x^5}$ (6)

c) Change the order of integration for the integral $\int_0^a \int_{y/2/a}^y f(x,y) dx dy$ (8)

Q.3 a) Evaluate $\int_0^3 e^{\sqrt{x}} dx$ by Simpson's 3/8 th rule. Take $h=0.25$. (6)

b) Apply Runge-Kutta method of fourth order to find an approximate value of y at $y=0.2$ if

$$\frac{dy}{dx} = x + y^2,$$

given that $y=1$ when $x=0$ in steps of $h=0.1$ (6)

c) Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$ (8)

Q.4 a) Solve $(D^3 + D)y = \sin x$ (6)

b) Solve $(x^3y^3 + xy)dy = dx$ (6)

c) Evaluate the integral $\int_0^2 \int_1^2 \int_0^{yz} xyz dx dy dz$. (8)

Q.5 a) Find $\int_0^1 \int_0^y xye^{-x^2} dx dy$ (6)

b) Evaluate $\int_0^2 y^4 (8 - y^3)^{-\frac{1}{3}} dy$ (6)

c) Solve (8)

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

Using the method of variation of parameters

Q.6 a) Solve $\frac{dy}{dx} \cosh x = 2(\cosh x)^2 \sinh x - y \sinh x$ (6)

b) a) Solve (6)

$$\frac{dy}{dx} = 2 + \sqrt{xy} \text{ with } x_0 = 1.2, y_0 = 1.6403 \text{ by Euler's modified formula for } x=1.4, x=1.6$$

c) Evaluate $\iint y dx dy$ throughout the area bounded by $x=0$, $y=x^2$ and $x+y=2$. (8)