Q. P. Code: 27175

(3 hours) Total Marks: 80

N.B. (1) Question no. 1 is Compulsory

(2) Solve any three from the remaining.

Q.(1)(a) If $5\sinh x - \cosh x = 5$ find $\tanh x$. (3)

(b) If $u = e^{x^2+y^2+z^2}$ prove that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = 8xyzu$. (3)

(c) If $u = \dfrac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$. (3)

(d) By Maclaurins series expand $\log\left(1+e^x\right)$ in powers of x upto x^4. (3)

(e) Show that the matrix $A = \dfrac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary and hence find A^{-1} (4)

(f) Find the n^{th} derivative of $y = \dfrac{x^2}{(x+2)(2x+3)}$ of (4)

Q.2) (a) Solve $x^5 = 1 + i$ and find the continued product of the roots. (6)

(b) Find the nonsingular matrices P and Q such that PAQ is in normal form also (6)

find the rank of A, where $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$

(c) State and prove Euler's theorem for homogeneous functions on three variables. (8)

Q.3) (a) Investigate for what values of λ and μ the equations (6)

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have i) no solutions. (6)

ii) a unique solution. iii) infinite number of solutions.

(b) Find the stationary valuesof $f(x,y) = x^3 + xy^2 + 21x - 12x^2 - 2y^2$ (6)

(c) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$ Prove that $\cos^4\theta = \sin^2\alpha = \sinh^4\phi$ (8)

Q.4) (a) If $z = e^{x/y} + \log\left(x^3 + y^3 - x^2y - xy^2\right)$ find the value of (6)

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2}.$$

(b) Show that $\tan^{-1} i\left(\frac{x-a}{x+a}\right) = \frac{i}{2}\log\frac{x}{a}$ (6)

(c) Solve the following equations by Gauss Jordan Method

$$2x + 3y + 4z = 11$$
$$x + 5y + 7z = 1$$
$$3x + 11y + 13z = 25$$

(8)

Q.5) (a) Find the value of a,b,c so that $\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$ (6)

(b) Expand $\log\left(1 + x + x^2 + x^3\right)$ upto x^8 (6)·

(c) If $y = \cos\left(m\sin^{-1}x\right)$ Prove that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$ (8)

Q.6) (a) Find a,b,c if A is orthogonal where $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ (6)

(b) Fit a second degree curve to the following data (6)

x	0	1	2	3	4
y	1.0	1.8	1.3	2.5	6.3

(c) If $x^x y^y z^z = c$ show that the value of $\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x\partial y} + \frac{\partial^2 z}{\partial y^2} = \frac{2\left(x^2 - 2\right)}{x\left(1 + \log x\right)}$, at x=y=z (8)
