Duration: 3hrs **[Max Marks:80]**

N.B. : (1) Question No 1 is Compulsory.
(2) Attempt any **three** questions out of the remaining five.
(3) All questions carry equal marks.
(4) Assume suitable data, if required and state it clearly.

1 (All questions of Q1 are compulsory) [20]

a Obtain $L\{f(3t)\}$ if $L\{f(t)\} = \frac{s^2-2s+7}{(s^3+5s^2-s+3)}$ [05]

b Obtain the Fourier series of $f(x) = x, \ -2 \le x \le 2$ [05]

c Find a, b, c, d if $f(z) = (ax^3 + bxy^2 + x) + i(3x^2y + cy^3 + dy)$ is analytic. [05]

d For the regression lines $2x - y + 5 = 0$ and $3y - x + 7 = 0$ find [05]
(i) the means of x and y and (ii) correlation coefficient r

2 a Find: $L^{-1}\{\frac{s}{(s^2+9)(s^2+1)}\}$ using convolution theorem. [06]

b Solve using Bender-Schmidt method: $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$; subject to the conditions: [06]
$u(0,t) = 0; \ u(3,t) = 0; \ u(x,0) = x^2(9-x^2)$
taking $h = 1$ upto 2 seconds

c Find orthogonal trajectories of $x^2 - y^2 - 2xy = c$ [08]

3 a Find the Complex form of Fourier series for $f(x) = e^x$ in $(0, 2\pi)$ [06]

b Obtain the Laurent's series for [06]
$f(z) = \frac{1}{(z+1)(z-4)}$ in $1 < |z+1| < 3$

c Obtain [08]
(i) $L\{e^t cos2t\}$ (ii) $L\{\int_0^t cos^2 2t \ dt\}$

4 a Find: $L^{-1}\{\frac{s+2}{s^2-4s+5}\}$ [06]

b Fit a straight line to the following data by the method of least squares where Y is the dependent variable: [06]

X	1	2	3	4	5	6
Y	11.7	14.6	13.2	15.4.	16.8	15.8

c Evaluate $\int_C \frac{1}{(z-1)(z-5)}\,dz$ where C is the circle $|z| = 2$ [08]

5 a Find an analytic function whose real part is $cos2xcosh2y$ [06]

b Solve using Crank-Nicolson formula: [06]

$\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0,\ 0 \le x \le 1$; subject to the conditions:

$u(0,t) = 0; u(1,t) = 0; u(x,0) = 100\,x\,(1-x)$ taking

$h = 0.25$ for one step

c Obtain the Half Range Fourier Cosine Series of [08]

$f(x) = x, 0 \le x \le 2$ and deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots$

6 a Obtain the Spearman's rank correlation coefficient of the following marks in Subjects X and Y : [06]

X	18	14	18	12	11	11
Y	15	20	16	10	10	15

b Obtain the Bilinear transformation that transforms the points $z = 1, 2, 0$ respectively to the points $w = 1, \infty, 2$ [06]

c Evaluate $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$ [08]
