S.E.(Mech)(Sem-IV)(CBCGS)(R2016) 92087

Applied Mathematics-IV

University of Mumbai

Examinations Summer 2022

Time: 2hour 30 minutes Max. Marks: 80

<table>
<tr><th>Q1.</th><th>Choose the correct option for following questions. All the Questions are compulsory and carry equal marks</th></tr>
<tr><td>1.</td><td>If $A = \begin{bmatrix}1 & 8\\2 & 1\end{bmatrix}$ then the eigenvalues of $A^2 + 2A + I$ are</td></tr>
<tr><td>Option A:</td><td>16, 36</td></tr>
<tr><td>Option B:</td><td>3, 5</td></tr>
<tr><td>Option C:</td><td>12,20</td></tr>
<tr><td>Option D:</td><td>12,36</td></tr>
<tr><td></td><td></td></tr>
<tr><td>2.</td><td>The matrix for $2A^5 - 3A^4 + A^2 - 4I$ where $A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$</td></tr>
<tr><td>Option A:</td><td>$130A - 430I$</td></tr>
<tr><td>Option B:</td><td>$135A - 403I$</td></tr>
<tr><td>Option C:</td><td>$138A - 403I$</td></tr>
<tr><td>Option D:</td><td>$138A + 403I$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>3.</td><td>The maximum directional derivative of $\varphi = x^2y^2z^4$ at $(3,-1,-2)$ is</td></tr>
<tr><td>Option A:</td><td>408.45</td></tr>
<tr><td>Option B:</td><td>418.45</td></tr>
<tr><td>Option C:</td><td>418.045</td></tr>
<tr><td>Option D:</td><td>408</td></tr>
<tr><td></td><td></td></tr>
<tr><td>4.</td><td>A random variable X has the probability distribution
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(X=x)</td><td>0.1</td><td>k</td><td>0.2</td><td>2k</td><td>0.3</td><td>k</td></tr>
</table>
The value of k is</td></tr>
<tr><td>Option A:</td><td>K=0.1</td></tr>
<tr><td>Option B:</td><td>K=0.2</td></tr>
<tr><td>Option C:</td><td>K=0.4</td></tr>
<tr><td>Option D:</td><td>K=0.15</td></tr>
<tr><td></td><td></td></tr>
<tr><td>5.</td><td>The divergence of $\bar{F} = 3x^2i + 5xyj + xyz^3k$ at the point $(1,2,3)$ is</td></tr>
<tr><td>Option A:</td><td>65</td></tr>
<tr><td>Option B:</td><td>60</td></tr>
<tr><td>Option C:</td><td>$27i - 18j + 10k$</td></tr>
<tr><td>Option D:</td><td>$27i + 18j + 10k$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>6.</td><td>A continuous random variable X has the following probability distribution $f(x) = \frac{3}{8}x^2; 0 < x < 2$ the $P(0.2 < X < 0.5)$is</td></tr>
<tr><td>Option A:</td><td>0.123</td></tr>
<tr><td>Option B:</td><td>0.321</td></tr>
<tr><td>Option C:</td><td>0.231</td></tr>
<tr><td>Option D:</td><td>0.0123</td></tr>
<tr><td></td><td></td></tr>
<tr><td>7.</td><td>A variable X follows Poisson distribution with variance 3, the $P(X \geq 4)$ is</td></tr>
<tr><td>Option A:</td><td>0.647</td></tr>
<tr><td>Option B:</td><td>0.353</td></tr>
</table>

Option C:	0.224
Option D:	0.546
8.	If the mean and variance of the binomial distribution are 3 and 1.2 respectively the value of n and p are
Option A:	n =5, p=0.6
Option B:	n=5, p=0.4
Option C:	n = 6, p=0.6
Option D:	n = 6, p=0.4
9.	The matrix form of quadratic equation $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_1x_2 + 18x_3x_1 + 4x_1x_2$
Option A:	$\begin{bmatrix} 6 & 2 & 9 \\ 2 & 3 & 2 \\ 9 & 2 & 14 \end{bmatrix}$
Option B:	$\begin{bmatrix} 6 & 4 & 18 \\ 4 & 3 & 4 \\ 18 & 4 & 14 \end{bmatrix}$
Option C:	$\begin{bmatrix} -6 & -2 & -9 \\ -2 & -3 & -2 \\ 9 & 2 & -14 \end{bmatrix}$
Option D:	$\begin{bmatrix} -6 & 4 & 18 \\ 4 & -3 & 4 \\ 18 & 4 & -14 \end{bmatrix}$
10.	The degree of freedom of the contingency table of the order 3x4 is
Option A:	12
Option B:	6
Option C:	8
Option D:	4

Q2	**Solve any Four out of Six5 marks each**
A	Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
B	Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$ to the canonical form by orthogonal transformation.
C	Find the directional derivatives of $\phi = xy^2 + yz^2$at the point (2,-1,1) in the direction of the vector $i + 2j + 2k$.
D	A sample of size 13 gave an estimated population variance 3.0, while another sample of size 15 gave an estimate of 2.5. Could both the samples be from population with the same variance?
E	On an average 20% of the population in an area suffer from T.B. What is the probability that out of 5 persons chosen at random from this area at least two suffer from T.B.
F	Using Big M Method/penalty method, solve the following LPP $Min.Z = 2x_1 + x_2$ Subject to the constraints $3x_1 + x_2 = 3$ $4x_1 + 3x_2 \geq 6$ $x_1 + 2x_2 \leq 3$ $x_1, x_2 \leq 0$
Q3	**Solve any Four out of Six5 marks each**

A If $A = \begin{bmatrix} \pi/2 & 3\pi/2 \\ \pi & \pi \end{bmatrix}$, find $cosA$.

B Write the dual of the primal is

$$Max.Z = 4x_1 + 5x_2 + 12x_3$$

Subject to

$$2x_1 + x_2 + x_3 \leq 4$$
$$3x_1 - 2x_2 + x_3 = 3$$
$$x_1, x_2, x_3 \geq 0$$

C Determine the constants a, b, c if $\bar{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational.

D A manufacturer knows from his experience that the resistance of resistors he produces is normal with mean 100ohms and standard deviation 2 ohms. What percentage of resistors will have resistance between 98 and 102 ohms?

E Samples of two types of electric bulbs were tested for length of life and the following data were obtained

	Type I	Type II
No. of samples	8	7
Mean of the samples (in Hrs.)	1134	1024
Standard Deviation	35	40

Test at 5% level of significance whether the difference in the sample means is significant.

F Evaluate by Stoke's theorem $\int_c (xydx + xy^2dy)$ where C is the square in the xy-plane with vertices $(1,0), (0,1), (-1,0) and (0,-1)$.

Q4 Solve any Four out of Six5 marks each

A Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Show that the matrix A satisfies the characteristic equation and hence find A^{-1}.

B Find the scalar potential if the $\bar{F} = (ysinz - sinx)i + (xsinz + 2yz)j + (xycosz + y2k$ is irrotational.

C Use Green's theorem to evaluate $\oint (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2$ and $y = 4$.

D Use the dual simplex method to solve the following LPP

$$Min.Z = 2x_1 + 2x_2 + 4x_3$$

Subject to the constraints

$$2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

E Based on the following data determine if there is a relation between literacy and smoking

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

F A continuous random variable has the following probability density function

$$f(x) = kx(1 - x); \ 0 \leq X \leq 1$$

Find i) k, ii) mean iii) variance