(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve $(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dy=0$ 5

b) Solve $(D^2-4D+4)y=e^{2x}+cos\,2\,x$ 5

c) Show that $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}}dx \ * \ \int_0^\infty y^4e^{-y^6}dy=\frac{\pi}{9}$ 5

d) Change the order of the following integration 5

$$I=\int_0^1\int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y)dydx$$

Q2.

a) Evaluate $I=\int\int\int\frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{3/2}}$, over the volume V bounded by the spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$, $(b>a)$ 6

b) Show that the length of the arc of the parabola $y^2=4ax$ from the vertex to the end of the latus rectum is $a[\sqrt{2}+\log(1+\sqrt{2})]$. 6

c) Solve by using method of variation of parameters 8

$$\frac{d^2y}{dx^2}+y=secx\,tanx$$

Q3.

a) Show that $\int_0^\pi \frac{log(1+a\cos x)}{\cos x}dx=\pi\,sin^{-1}a\,,0\le a\le 1.$ Hence evaluate $\int_0^\pi\frac{log(1+\cos x)}{\cos x}\ dx$ 6

b) Evaluate $I = \iint y^2 dxdy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$. 6

c) Evaluate $I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 yz \, dxdydz$ 8

Q4.

a) Solve $\cosh x \frac{dy}{dx} = 2\cosh^2 x \sinh x - y \sinh x$ 6

b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ 6

c) Show that $\int_0^\infty \frac{dx}{(e^x + e^{-x})^n} = \frac{1}{4}\beta\left(\frac{n}{2}, \frac{n}{2}\right)$, hence find the value of $\int_0^\infty sech^6 x \, dx$ 8

Q5.

a) Evaluate $I = \int_{-1}^{1} \int_0^{1-x} x^{1/3} \ y^{-1/2} \left((1 - x - y)^{1/2}\right) dxdy$ 6

b) Find the area inside the circle $r = a$ and outside the cardioide $r = a(1 + \cos\theta)$ 6

c) Solve $xy\,(1 + xy^2)\frac{dy}{dx} = 1$ 8

Q6.

a) Solve $(D^2 + 2)y = x^2 e^{3x} + \ x \sin 3x$ 6

b) Solve $xe^x(dx - dy) + e^x dx + ye^y dy = 0$ 6

c) Change the order of integration and evaluate 8

$$I = \int_0^a \int_0^x \frac{dxdy}{(y+a)\sqrt{(a-x)(x-y)}}$$

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