

(Time: 3 hours)

Max.Marks:80

- N.B
- (1) Question No.1 is compulsory
 - (2) Answer any three questions from Q.2 to Q.6
 - (3) Use of Statistical Tables permitted
 - (4) Figures to the right indicate full marks.

- 1
 - a) Prove that $\sec h^{-1}(\sin \theta) = \log \left(\cot \frac{\theta}{2} \right)$ 5
 - b) If $z = x^y + y^x$ then prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ 5
 - c) If α, β are the roots of the quadratic equation $x^2 - 2\sqrt{3}x + 4 = 0$, find the value of $\alpha^3 + \beta^3$ 5
 - d) Test the consistency and if possible solve $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$ 5
- 2
 - a) Is $A = \begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ a unitary matrix? 6
 - b) Find the n^{th} derivative of $y = \frac{4x}{(x-1)^2(x+1)}$ 6
 - c) If $u = \frac{x^4 + y^4}{x^2 y^2}$ then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ at $x = 1$ and $y = 2$ 8
- 3
 - a) Prove that $\log (1 + \cos 2\theta + i \sin 2\theta) = \log (2 \cos \theta) + i\theta$ 6
 - b) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ using De Moivre's theorem 6
 - c) Discuss for all values of k for which the system of equations has a non-trivial solution 8

$2x + 3ky + (3k + 4)z = 0,$
 $x + (k + 4)y + (4k + 2)z = 0, \quad x + 2(k + 1)y + (3k + 4)z = 0$

4 a) If $u = \log(r)$ and $r = x^3 + y^3 - x^2y - xy^2$ then show that 6

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$$

b) Find two non-singular matrices P and Q such that PAQ is in 6
 the normal form where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

c) Prove that $\tan^{-1}(e^{i\theta}) = \frac{n\pi}{2} + \frac{\pi}{4} - \frac{i}{2} \log \tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right)$ 8

5 a) Considering principal value, express in the form $a + ib$ the 6
 quantity $(\sqrt{i})^{\sqrt{i}}$

b) Prove that $\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ 6

c) If $y = e^{a \sin^{-1} x}$, then Prove that 8
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ Also find $y_n(0)$

6 a) If $u = \frac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ 6

b) If $\frac{3}{x} + \frac{4}{y} + \frac{5}{z} = 6$ find the values of x, y, z such that $x + y + z$ is 6
 minimum

c) Prove that every Skew-Hermitian matrix can be expressed in 8
 the form $B+iC$, where B is real Skew-Symmetric and C is real Symmetric matrix and express the matrix

$$A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix} \text{ as } B+iC \text{ where B is real Skew-symmetric matrix and C is real Symmetric matrix}$$