Q.P. Code :16282

Max. Marks: 80 **Time: 3 hrs.**

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.

Q1. a) Evaluate $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx$ 3

b) Solve $\left(D^3+1\right)^2 y = 0$ 3

c) Solve the ODE $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\left(x+xy^2\right)dy=0$ 3

d) Use Taylor's series method to find a solution of $\frac{dy}{dx}=1+y^2,\quad y(0)=0$ 3

at $x=0.1$ taking $h=0.1$ correct to three decimal value.

e) Given $\int_0^x \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$, using DUIS find the value of $\int_0^x \frac{dx}{\left(x^2+a^2\right)^2}$ 4

f) Find the perimeter of the curve $r=a(1-\cos\theta)$. 4

Q2. a) Solve $\left(D^3+D^2+D+1\right)y=\sin^2 x$ 6

b) Change the order of integration $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y)\,dx\,dy$ 6

c) Evaluate $\iint_R \frac{2xy^5}{\sqrt{1+x^2y^2-y^4}}\,dx\,dy$, where R is a triangle whose vertices 8

are $(0,0),(1,1),(0,1)$.

Q3. a) Find the volume enclosed by the cylinder $y^2=x$ & $y=x^2$ 6

cut off by the planes $z=0,\ x+y+z=2$.

b) Using Modified Euler's method, find an approximate value of y at 6

$x=0.2$ in two step taking $h=0.1$ and using three iteration, given that

$\frac{dy}{dx}=x+3y,\quad y=1$ when $x=0$.

c) Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x)$ 8

Q4. a) Show that $\int_0^a \sqrt{\frac{x^3}{a^3-x^3}}\,dx=\frac{a\sqrt{\pi}\,\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{1}{3}\right)}$ 6

b) Solve $\left(D^2+2\right)y=e^x\cos x+x^2e^{3x}$ 6

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c) Use polar co-ordinates to evaluate $\iint \frac{\left(x^2+y^2\right)^2}{x^2 y^2}\, dx\, dy$ over the area common to the circle $x^2+y^2=ax$ and $x^2+y^2=by$, a > b > 0. 8

Q5. a) Solve $y\,dx + x\left(1-3x^2y^2\right)dy = 0$ 6

b) Find the mass of a lamina in the form of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if the density at any point varies as the product of the distance from the axes of the ellipse. 6

c) Compute the value of $\int_0^{\pi/2} \sqrt{\sin x + \cos x}\, dx$ using (i) Trapezoidal rule (ii) Simpson's $(1/3)^{rd}$ rule (iii) Simpson's $(3/8)^{th}$ rule by dividing into six subintervals. 8

Q6. a) Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the volume bounded by the planes $x=0,\ y=0,\ z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ 6

b) Change the order of integration and evaluate $\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2}{\sqrt{x^4-4y^2}}\, dx\, dy$ 6

c) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ 8

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