Paper / Subject Code: 38901 / APPLIED MATHEMATICS - IV Date-4/12/19

S.E. (Computer) (Sem-IV) (CBSGS) (R-2012)

Time Duration: 3Hr **Total Marks: 80**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Use of statistical tables permitted.
4) Figures to the right indicate full marks.

Q1. a) Evaluate $\int_C (z - z^2)dz$, where C is the upper half of circle $|z| = 1$. **[5]**

b) If A=$\begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$, find the Eigen values of $A^2 - 2A + I$. **[5]**

c) State whether the following statement is true or false with reasoning: "The line of regression between x and y are parallel to the line of regression between 2x and 2y." **[5]**

d) Find the dual of the following L.P.P. **[5]**

Maximize $z = 3x_1 + 17x_2 + 9x_3$

Subject to $x_1 - x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 = 1$

$x_1, x_2, x_3 \geq 0$

Q2. a) Evaluate $\int_C \frac{1}{z^3(z+4)} dz$, where c is the circle $|z|=2$. **[6]**

b) Show that the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is non-derogatory. **[6]**

c) For a normal variate X with mean 2.5 and standard deviation 3.5, find the probability that (i) $2 \leq X \leq 4.5$, (ii)$-1.5 \leq X \leq 5.3$. **[8]**

Q3. a) Find the expectation of number of failures preceding the first success in an infinite series of independent trials with constant probabilities p and q of success and failure respectively. **[6]**

b) Solve the following L.P.P. by simplex method **[6]**

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

c) Expand $f(z) = \frac{2-z^2}{z(1-z)(2-z)}$ about $Z = 0$ indicating the region of convergence in each case. **[8]**

Q4. a) A biased coin is tossed n times. Prove that the probability of getting even number of heads is $0.5[1 + (q - p)^n]$. **[6]**

b) Calculate the coefficient of correlation between X and Y from the following data. **[6]**

X	100	200	300	400	500
Y	30	40	50	60	60

c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming matrix M and the diagonal form D. **[8]**

Q5.a) Can it be concluded that the average life-span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation of 8.9 years? **[6]**

b) Evaluate $\int_0^\infty \frac{1}{x^4+1}\,dx$, using Cauchy's residue theorem. **[6]**

c) Using the Kuhn – Tucker conditions, solve the following N.L.P.P. **[8]**
Minimize $z = 7{x_1}^2 + 5{x_2}^2 - 6x_1$
Subject to $x_1 + 2x_2 \le 10$
$x_1 + 3x_2 \le 9$
$x_1, x_2 \ge 0$

Q6.a) A die was thrown 132 times and the following frequencies were observed. **[6]**

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

b) If two independent random samples of sizes 15 and 8 have respectively the following means and population standard deviations, **[6]**
$\overline{X_1} = 980 \quad \overline{X_2} = 1012$
$\sigma_1 = 75 \quad \sigma_2 = 80$
Test the hypothesis that $\mu_1 = \mu_2$ at 5% level of significance.

b) Using Penalty (Big-M) method solve the following L.P.P. **[8]**
Maximise $z = 3x_1 - x_2$
Subject to $2x_1 + x_2 \le 2$
$x_1 + 3x_2 \ge 3$
$x_2 \le 4$
$x_1, x_2 \ge 0$
