Time: 3 hour **Max. Marks: 80**

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1	**Attempt All questions**	**Marks**
a)	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 1 \end{bmatrix}$ then find the eigen values of A^3	**5**
b)	Find Laplace transform of $f(t) = te^t \cos 2t$	**5**
c)	Find the Fourier Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$	**5**
d)	Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$is analytic.	**5**
Q2		
a)	A vector field $\bar{F}$ is given by $\bar{F} = (ysinz - sinx)i + (xsinz + 2yz)j + (xycosz + y^2)k$ Prove that $\bar{F}$ is irrotational.	**6**
b)	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	**6**
c)	Show that the function u = sinx coshy + 2cosx sinhy + $x^2 - y^2$ + 4xy satisfies Laplace's equation, also find analytic function.	**8**
Q3		
a)	If $\bar{F} = xye^{2z}i + xy^2coszj + x^2cosxyk$ find div$\bar{F}$ and curl$\bar{F}$	**6**
b)	Find an analytic function whose real part is u = $y^3 - 3x^2y$. Also find the corresponding imaginary part.	**6**
c)	Show that the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable and hence find the transforming matrix and diagonal matrix.	**8**

Q4

a) Find $\nabla\phi$ at point (1, -2, -1), where $\phi = 4xz^2 + x^2yz$ **6**

b) Evaluate $\int_0^\infty e^{-2t} sin^3 t \ dt$, using Laplace transforms **6**

c) Using Partial Fraction method find $L^{-1}\left[\frac{s}{(s^2+1)\,(s^2+4)\,(s^2+9)}\right]$ **8**

Q5

a) Find $L\left\{t\sqrt{1+sint}\right\}$ **6**

b) Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by **6**

$$\bar{F}(x,y,z) = y\,\hat{\boldsymbol{\imath}} + (zcos(yz)+x)\,\hat{\boldsymbol{\jmath}} + (y\cos(yz))\,\hat{\boldsymbol{k}}$$

Show that $\bar{F}$ is conservative.

c) Find the Fourier Series for $f(x)$ in $(-\pi,\pi)$where **8**

$$f(x) = 1+\frac{2x}{\pi} \qquad -\pi \le x \le 0$$
$$= 1-\frac{2x}{\pi} \qquad 0 \le x \le \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$

Q6

a) Obtain Fourier series expansion of $f(x) = 9 - x^2$ in $(0, 2\pi)$ **6**

b) Find Eigen values and Eigen vectors of **6**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

c)

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{(s+a)}{(s+b)}}\right)\right\}$ **4**

ii) Find $L^{-1}\left\{\frac{1}{s^2-2s+5}\right\}$ **4**