

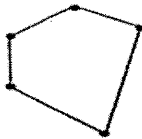
(3 hrs)

Max. Marks: 80

- 1) Question no.1 is compulsory.
- 2) Solve any **THREE** questions out of remaining **FIVE** questions.
- 3) All questions carry equal marks as indicated by figures to the right.
- 4) Assume appropriate data whenever required. State all assumptions clearly.

- Q.1 a) Prove using Mathematical Induction (05M)
 $2+5+8+\dots+(3n-1)=n(3n+1)/2$
- b) Find the generating function for the following finite sequences (05M)
 i) 1,2,3,4,... ii) 2,2,2,2,2
- c) Let $A = \{1, 4, 7, 13\}$ and $R = \{(1,4), (4,7), (7,4), (1,13)\}$ (05M)
 Find Transitive Closure using Warshall's Algorithm (05M)
- d) Let $f: R \rightarrow R$, where $f(x) = 2x - 1$ and $f^{-1}(x) = (x+1)/2$ (05M)
 Find $(f \circ f^{-1})(x)$

- Q.2 a) Define Lattice. Check if the following diagram is a lattice or not. (04M)



- b) Prove that set $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication module 7 (08 M)
- c) A travel company surveyed it's travelers, to learn how much of their travel is taken with an Airplane, a Train or a Car. The following data is known; make a complete Venn Diagram with all the data. The number of people who flew was 1307. The number of people who both flew and used a train was 602. The people who used all three were 398 in number. Those who flew but didn't drive came to a total of 599. Those who drove but did not use a train totaled 1097. There were 610 people who used both trains and cars. The number of people who used either a car or a train or both was 2050. Lastly, 421 people used none of these Find out how many people drove but used neither a train nor an airplane, and also, how many people were in the entire survey. (08 M)

- Q.3 a) Prove $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. (04 M)

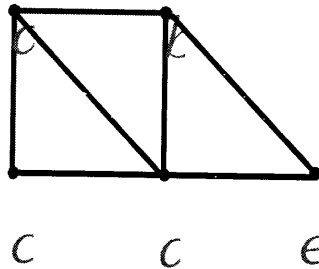
- b) Consider the (3,5) group encoding function defined by (08 M)

$e(000)=00000$	$e(001)=00110$
$e(010)=01001$	$e(011)=01111$
$e(100)=10011$	$e(101)=10101$
$e(110)=11010$	$e(111)=11000$

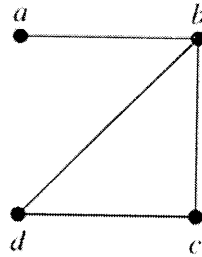
Decode the following words relative to a maximum likelihood decoding function.

- i) 11001 ii) 01010 iii) 00111
- c) Mention all the elements of set D_{36} also specify R on D_{36} as aRb if $a \mid b$. Mention Domain and Range of R . Explain if the relation is Equivalence Relation or a Partially Ordered Relation. If it is a Partially Ordered Relation, draw its Hasse Diagram. (08 M)

- Q.4 a) Explain Extended pigeonhole Principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. (04 M)
- b) Define Euler Path and Hamiltonian Path.
- i) Determine Euler Cycle and path in graph shown in (a)
- ii) Determine Hamiltonian Cycle and path in graph shown in (b)



(a)



(b)

- c) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there? (08 M)
- Q.5 a) Let G be a group. Prove that the identity element e is unique. (04M)
- b) A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen? (08M)
- c) Let A be a set of integers, let R be a relation on $A \times A$ defined by $(a,b) R (c,d)$ if and only if $a+d=b+c$. Prove that R is an equivalence Relation. (08M)

- Q.6 a) Define reflexive closure and symmetric closure of a relation. Also find reflexive and symmetric closure of R . (04 M)

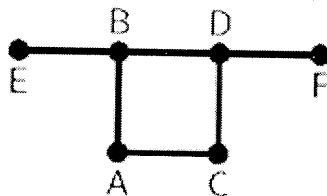
$$A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$$

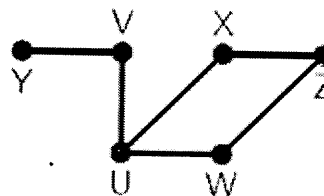
b) Let $H =$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

- Be a parity check matrix. Determine the group code $e_H: B^3 \rightarrow B^6$ (08M)
- c) Determine if following graphs G_1 and G_2 are isomorphic or not. (08M)



G_1



G_2

Q. 4 (b)

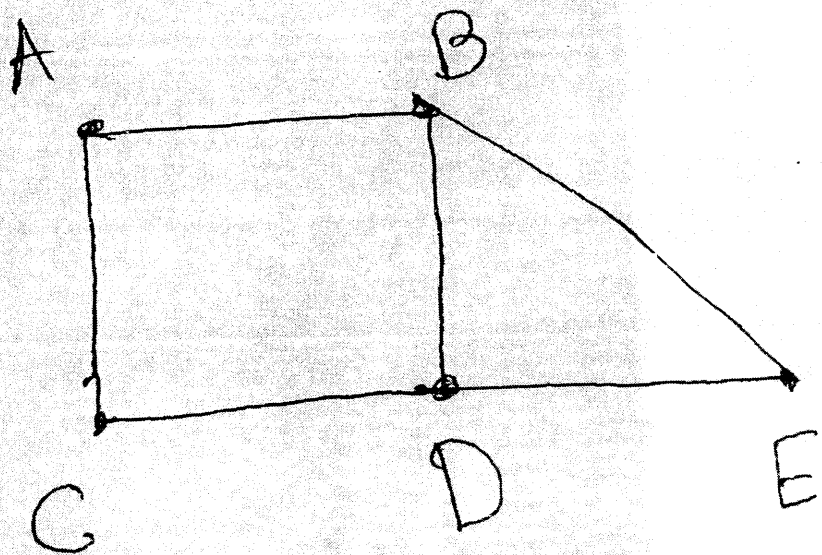


fig (a)