

(3 hours)

Total Marks-80

N.B. 1) Question No.1 is compulsory.

2) Attempt any THREE questions from Q.No.2 to Q.No.6

3) Figures to the right indicate full marks

- Q1 a) Find $L\left[\frac{\cos 2t \sin t}{e^t}\right]$ [5]
- b) Determine the constants a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. [5]
- c) Find Half range cosine series for $f(x) = x(\pi - x), 0 < x < \pi$ [5]
- d) Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point(2,-1,1) in the direction of the vector $i + 2j + 2k$ [5]
- Q2) a) Show that the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. [6]
Find its harmonic conjugate and corresponding analytic function.
- b) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1,1)$. [6]
- c) Find i) $L^{-1}\left[\frac{e^{-\pi s}}{s^2 - 2s + 2}\right]$ [8]
ii) $L^{-1}\left[\tan^{-1}\left(\frac{s+a}{b}\right)\right]$
- Q3) a) Find the angle between the surfaces $x \log z + 1 - y^2 = 0$, [6]
 $x^2y + z = 2$ at (1,1,1)
- b) Prove that $J'_2(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ [6]

- c) Obtain Fourier series for [8]

$$f(x) = \begin{cases} x + \frac{\pi}{2} & , -\pi < x < 0 \\ \frac{\pi}{2} - x & , 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \dots$

- Q4) a) Using Gauss's Divergence theorem, prove that [6]

$\iint_S (y^2 z^2 i + z^2 x^2 j + x^2 y^2 k) \cdot \bar{N} ds = \frac{\pi}{12}$ where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy- plane.

- b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$ [6]

- c) Solve using Laplace Transform $(D^2 + 2D + 5)y = e^{-t} \sin t$, [8]
when $y(0) = 0, y'(0) = 1$

- Q5) a) Find inverse Laplace Transform using convolution theorem for [6]

$$\frac{1}{(s-a)(s+a)^2}$$

- b) Prove that $J_3(x) + 3J_0(x) + 4J_0'''(x) = 0$ [6]

- c) Obtain the complex form of Fourier Series for $f(x) = e^{ax}$ in $(-l, l)$ [8]

- Q6) a) Using Green's Theorem in the plane evaluate [6]

$\oint (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = x^2, y = 4$

- b) Show that the map of real axis of the Z-plane is a circle under the [6]
transformation $w = \frac{2}{z+i}$. Find its centre and the radius.

- c) Find Fourier Integral Representation for [8]

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$
