Duration: 3 hours   **Max. Marks: 80**

N.B. (1) Question No. 1 is **COMPULSORY**.

  (2) Answer **ANY THREE** questions from Q.2 to Q.6.

  (3) Use of Statistical Tables permitted.

  (4) Figures to right indicate full marks.

Que. 1   a.   Evaluate the Complex line integral $\int_c (y - x - 3x^2 i)dz$,   **5**

Where C is the straight line from z=0 to z= 1+i.

b.   Find a vector orthogonal to both $u = (-6, 4, 2), v = (3, 1, 5)$ **5**

c.   Calculate the coefficient of correlation between by X and Y   **5**

X	3	5	4	6	2
Y	3	4	5	2	6

d.   The Probability function of a random variable X is given by   **5**

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Find (i) k   (ii) mean   (iii) variance

Que. 2   a.   Find the extremals   of $\int_0^1 (xy + y^2 - 2y^2 y') \, dx$.   **6**

b.   Monthly salary X in a big organization is normally distributed with mean   **6**
Rs. 3000 and standard deviation of Rs. 250. What should be the minimum
salary of a worker in this organization, so that the probability that he belongs
to top 5% workers?

c.   Find all possible Laurent's expansions of the function $f(z) = \frac{2}{(z-1)(z-2)}$ **8**
indicating the region of convergence

Que. 3   a.   A random variable X has the following density function   **6**

$$f(x) = \begin{cases} \frac{x}{6} + kx & 0 \le x \le 3 \\ 0 & , \ otherwise \end{cases},$$

find k and $P(1 \le x \le 2)$.

b.   Evaluate   **6**

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} \, dz \text{ , where C is the circle } |z| = 3 \text{ .}$$

c.   Reduce the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2 x_3 - 4x_3 x_1 + 12x_1 x_2$ to   **8**
normal form through congruent transformations. Also find it's rank,
signature and value class.

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Que. 4 a. Construct an orthonormal basis of R^3 using Gram-Schmidt process to **6**

$$S = \{(3, 0, 4), (-1, 0, 7), (2, 9, 11)\} \ .$$

      b. Find the Probability that at most 5 defective diodes will be found in pack of **6**
600 diodes ; if previous data shows that 3 % of each such diodes are
defective

      c. Solve by Rayleigh-Ritz method the boundary value problem **8**

$$I = \int_0^1 (2xy - y^2 - y'^2) \, dx \text{ given } \quad y(0) = 0 \text{ and } y(1) = 0 \ .$$

Que. 5 a. Find the spearman's rank of correlation for the following data **6**

          X : 12 17 22 27 32

          Y : 113 119 117 115 121 .

      b. Using Cauchy's Residue theorem evaluate $\oint_C \frac{e^{2z}}{(z-\pi i)^3} \, dz$ where C is **6**
$|z - 2i| = 4$.

      c. Find the singular value decomposition of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ **8**

Que. 6 a. Verify Cauchy-Schwartz Inequality for the vectors **6**
$u = (-4, 2, 1) \text{ and } v = (8, -4, -2)$.

      b. Determine whether the set of vectors of the form (a, b, c) where $b = a +$ **6**
c form a subspace of R^3 under usual addition and scalar multiplication

      c. Obtain the equations of the line of regression for the following data. **8**

      X : 5 6 7 8 9 10 11

      Y : 11 14 14 15 12 17 16