S.E. (Computers) (Sem-III) (CBSGS) (R-2012)

Paper / Subject Code: 49305 / DISCRETE STRUCTURE

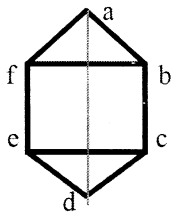
Date - 26/11/19

(3 Hours) **[Total marks: 80]**

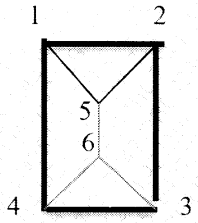
N.B (1) Question No. 1 is compulsory.
(2) Solve any three questions out of remaining five questions.
(3) Assumptions made should be clearly stated.
(4) Figures to the right indicate full marks.

Q.1 (a) For all sets A,X and Y show that [6M]
$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

(b) Define isomorphic graphs. Determine whether the following graphs are isomorphic or not. Justify your answer. [6M]



Fig(a)



Fig(b)

(c) Functions f ,g, h are defined on a set, [8M]

X={ 1,2,3} as
f = {(1, 2) ,(2,3) (3,1) }
g = {(1, 2), (2,1) (3,3) }
h = {(1, 1) ,(2,2) (3,1) }

(i) Find f o g , g o f, Are they equal ?
(ii) Find f o g o h and f o h o g

Q.2 (a) How many integers between 1 and 1000 are
i) not divisible by 3, nor by 5,nor by 7?
ii) not divisible by 5 and 7 but divisible by 3? [6M]

(b) State pigeonhole principle and extended pigeonhole principle. What is the minimum number of students required in a discrete structures class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C ,D and E. [6M]

(c) Solve $a_{r+2} - a_{r+1} - 6a_r = 4$ [8M]

Q.3 (a) Prove by mathematical induction. [6M]

$$1 + a + a^2 + \ldots\ldots\ldots + a^n = \frac{1 - a^{n+1}}{1 - a}, \text{ where } n \geq 0$$

(b) If (G, *) is an Abelian group, then for all a, b $\in$ G show that $(a * b)^n = a^n * b^n$ [6M]

(c) Let $a_r = 3^r$ $r \geq 0$ and [8M]
$b_r = 3^r$ $r \geq 0$ Find c_r that is $a_r * b_r$

Q.4 (a) Show that, $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \leftrightarrow R$ [6M]

(b) Define Hamitonian circuit and Hamitonian path. Determine which of the following graph contain an Eulerian or Hamitonian circuit.If it does, find such circuit. [6M]

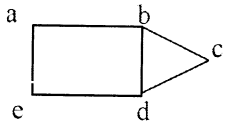


Fig (a)

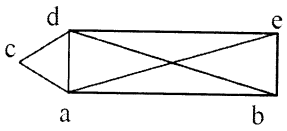


Fig (b)

(c) For each of the following sets of weights construct an optimal binary prefix code. For each weight in the set give the corresponding code word: [8M]

(1) 1,2,4,5 6,9,10,12
(2) 10,11,14,16,18,21

Q.5 (a)Define (i) Bounded Lattice (ii)Distributive Lattice (iii) Complemented Lattice [6M]

(b) Define planar graph. Determine the number of regions defined by a connected planar graph with 6 vertices and 10 edges. [6M]

(c) Consider (3,7) encoding function e: $B^3 \rightarrow B^7$ defined by [8M]

e (000)=0000000 e(100)=1000101
e (001)=0010110 e(101)=1010011
e (010)=0101000 e(110)=1101101
e (011)=0111110 e(111)=1111011

is a group code. How many errors it can detect?

Q.6 (a) Determine whether the given relation R is an equivalence relation.
R = {(6,6),(4,4),(3,3),(4,6),(6,4)(3,6)(6,3)(4,3)(3,4)} [6M]

(b) Determine whether the set together with the binary operation * is a semigroup ,monoid or a group .Justify your answer. [6M]

(i) Set of real numbers with a*b = a+b+2
(ii) The set of m × n matrices under the operation of multiplication.

(c) Give Prim's algorithm for minimum spanning tree. Use the same to find a minimum tree for graph shown in the following figure. [8M]

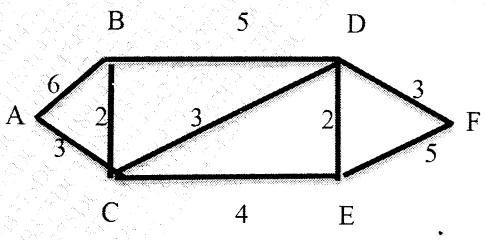


Fig (a)

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