

[Time: 3 Hours]

[ Marks:80]

Please check whether you have got the right question paper.

- N.B:** 1. Q 1 is compulsory.  
 2. Attempt any three from remaining  
 3. Rights indicate full marks.

1. a. If A, B, C are subset of universal set V then prove that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$  **05**  
 b. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $y = 2x + 1$ , prove that f is one to one and onto and find  $f^{-1}$  **05**  
 c. Find  $L\{(1 + te^t)^3\}$  **05**  
 d. Check whether the following function Harmonic or not  $3x^2 + \sin x + y^2 + 5y + 4$  **05**
2. a. Find k if  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{ky}{x}$  is analytic **06**  
 b. Find  $L\{\sin 2t\}$  **06**  
 c. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$   $f(x) = x^2 + 2x - 1$  **08**  
 $g: \mathbb{R} \rightarrow \mathbb{R}$   $g(x) = 4x^2 + 2$   
 Find (I)  $f \circ (g \circ f)$  (II)  $g \circ (f \circ g)$
3. a. Find Bilinear transformation under which  $Z=1, -i, -1$  from point  $w=i, 0, -i$  **06**  
 b. If A be the set of non-integers and let R be a relation on  $A \times A$  defined by (a, b) R (c, d) if  $ad=bc$ , then prove that R is an equivalence relation. **06**  
 c. Find (1)  $L\left\{\int_0^t e^{-u} \frac{\sin u}{u} du\right\}$  **08**  
 (2)  $L\{(1 + 2t + 3t^2 + t^3)H(t - 2)\}$
4. a. Use convolution theorem and evaluate **06**  

$$L^{-1}\left\{\frac{(s+5)^2}{(s^2+10s+16)^2}\right\}$$
  
 b. Find transitive closure of following relation defined on  $A = \{a, b, c, d, e\}$  by Warshall algorithm  $R = \{(a, a) (a, b) (b, c) (c, d) (c, c) (d, e)\}$  **06**  
 c. A man speaks truth 3 times out of 5 when a die is thrown he states that it gave an ace what is probability that this event has actually happened. **08**

5. a. How many four digit numbers can be formed out of the digits 1, 2, 3, 5, 7, 8, 9 if no digit is repeated twice? How many of them will be greater than 3000. 06
- b. Solve using Laplace transform 06  
 $\frac{d^2y}{dt^2} + 9y = 18$  given that  $y(0) = 0$  and  $y(\frac{\pi}{2}) = 0$
- c. Evaluate (1)  $L^{-1} \left\{ \frac{1}{\sqrt{2s+1}} \right\}$  08  
 (2)  $L^{-1} \left\{ \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6} \right\}$
6. a. Solve  $a_n = 5a_{n-1} - 6a_{n-2}$  for  $n \geq 2, a_0 = 0, a_1 = 1$  06
- b. Find orthogonal curves of family of curves  $e^{-x} \cos y + xy = \alpha$ , where  $\alpha$  is the real constant 06
- c. i. Find the image of rectangular region bounded by  $x=0, x=3, y=0, y=2$  under the transformation  $w = z + (1+i)$  08  
 ii. A fair dice is thrown thrice. Find probability that sum of numbers obtained is 10.

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