(3 Hours) **[Total Marks: 80]**

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q 1. a) Evaluate the Laplace transform of $L[(\sin 2t - \cos 2t)^2]$ [5]

b) Determine the constants a, b, c, d so that the function $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is analytic [5]

c) If $\phi = 3x^2y - y^3z^2$ find $\nabla \phi$ at the point P (1,-2,-1) [5]

d) Obtain half range sine series for f (x) = x^2 in 0 < x < 3 [5]

Q 2. a)Construct analytic function whose real part is $e^x \cos y$ [6]

b) Find the Fourier series for $f(x)$ = | x | in $(-2, 2)$. [6]

c) Find the Laplace transform of the following

i) $L[t\sqrt{1+\sin t}]$ ii) $L\left\{\frac{\sin t \sin 5t}{t}\right\}$ [8]

Q 3. a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ [6]

b) Evaluate inverse Laplace transform using Convolution Theorem $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ [6]

c) Show that the vector field $\overline{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + (3xz^2 + 2z)\hat{k}$ is conservative and find $\phi(x, y, z)$ such that $\overline{F} = \nabla \phi$. [8]

Q 4 a)Find bilinear transformation which maps the points z=0,i,-2i of z plane onto the points $w = -4i, \infty, 0$ of w plane [6]

b) Prove that $f_1(x) = 1,\ f_2(x) = x,\ f_3(x) = \frac{3x^2 - 1}{2}$ are orthogonal over $(-1,1)$. [6]

c) Find the Fourier transform of $f(t) = e^{-|t+1|}$ [8]

Q 5 a) Solve Using Laplace transform $\frac{d^2y}{dt^2} - 4y = 3e^t$ where $y(0) = 0$ & $y'(0) = 3$ [6]

b) Find Complex form of the Fourier series for $f(x) = e^{ax}$ in $-\pi < x < \pi$ [6]

c) Verify Green's Theorem for $\oint_C 2y^2\, dx + 3xdy$ where C is the boundary of the closed region bounded by $y = x^2$ and $y = x$. [8]

Q 6. a) Evaluate $L^{-1}\left[\dfrac{se^{-\frac{s}{2}} + \pi e^{-s}}{(s^2 + \pi^2)}\right]$ [6]

b) Find the map of the line x-y=1 by transformation $w = \dfrac{1}{z}$ [6]

c) Using Stoke's theorem evaluate $\oint_C (y\,dx + z\,dy + x\,dz)$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and plane $x + z = a$ [8]

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