S.E. (EXTC) (Sem-IV) (CBS) Date-4/12/19

**(3 Hours)** **[Total Marks: 80]**

N.B.: 1) Question No. 1 is Compulsory.
2) Answer any THREE questions from Q.2 to Q.6.
3) Figures to the right indicate full marks.

1) a) If $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ then find the eigen values of $6A^{-1} + A^3 + 2I$ [05]

b) Determine whether the given vectors u= (-4,6,-10,1),v= (2,1,-2,9) are orthogonal with respect to the Euclidean inner product [05]

c) The probability density function of a random variable x is zero except at x =0, I, 2 and

$p(0) = 3\alpha^3$, $p(1) = 4\alpha - 10\alpha^2$, $p(2) = 5\alpha - 1$. Find $\alpha$ [05]

d) Evaluate $\oint_c \frac{z+6}{z^2-4} dz$ where c is (i) $|z| = 1$ (ii) $|z-2| = 1$. [05]

2) a) Using Rayleigh-Ritz method, find an appropriate solution for the extremal of the functional

$I = \int_0^1 \left[ 2xy - y^2 - y'^2 \right] dx$ given y(0)=y(1)=0 [06]

b) Using Cauchy's Residue theorem evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ [06]

c)A random variable X has the probability distribution given below:

| X=x | -2 | 3 | 1 |
|---|---|---|---|
| P(X=x) | 1/3 | 1/2 | 1/6 |

Find i) the moment generating function ii) the first four moments about the origin [08]

3) a) Compute $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ [06]

b) Verify Cauchy –Schwartz inequality for the vectors u=(-4,2,1) &v=(8,-4,-2) [06]

c) Obtain Taylor's or Laurent's series expansion of the function $f(z) = \frac{1}{z^2 - 3z + 2}$ when

(i) $|z| < 1$ (ii) $1 < |z| < 2$ [08]

4) a)Obtain the equation of the line of regression of Y on X for the following data and estimateY when X = 73 [06]

| X | 70 | 72 | 74 | 76 | 78 | 80 |
|---|---|---|---|---|---|---|
| y | 163 | 170 | 179 | 188 | 196 | 200 |

b)Show that the functional $\int_{x_1}^{x_2} [y^2 + x^2 y'] dx$ assumes extreme values on the straight line y = x [06]

c)Let $R^3$ have the Euclidean inner product.Use the Gram-Schmidt process to transform the basis vectors $u_1=(1,0,0), u_2=(3,7,-2), u_3=(0,4,1)$ in to an orthonormal basis [08]

5) a)Evaluate $\int_c \frac{1}{z} \cos z \, dz$ where c is the ellipse $9x^2 + 4y^2 = 1$ [06]

b) Seven dice are thrown 729 times. How many times do you expectat leastfour 10 dice to show three or five? [06]

c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagnosable. Find the diagonal form D and the diagonalaising matrix M. [08]

6) a) A continuous random variable X has the p.d.f. defined by $f(x)=A+Bx, 0 \le x \le 1$. If the mean of the distribution is $\frac{1}{3}$ find A and B [06]

b) Find $e^A$, if $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ [06]

c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)} \quad (a > 0, b > 0)$ [08]

**********



0FEF08902C0543F1E6078AB447026784