Paper / Subject Code: 51401 / Applied Mathematics-III

S. E. (IT) (Sem-III) (CB) Date- 8/5/19

Total Marks: 80 **Hours: 3 hrs**

Note : 1) Question no. 1 is compulsory.
2) Attempt any three questions out of five questions

Q-1

a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiplies of each other. **(05)**

b) A function $f : R-\left\{\frac{7}{3}\right\} \to R-\left\{\frac{4}{3}\right\}$ is defined by $f(x)=\frac{4x-5}{3x-7}$, Prove that f is bijective and find the rule for f^{-1}. **(05)**

c) Find $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$ **(05)**

d) Prove that there does not exist an analytic function whose imaginary part is $3x^2+\sin x+y^2+5y+4$. **(05)**

Q-2

a) Find $L^{-1}\left[\frac{s}{(s^2+3^2)(s^2+5^2)}\right]$ using convolution Theorem. **(06)**

b) What is the chance of throwing ten with four dice? **(06)**

c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he gives answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. **(08)**

Q-3

a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? **(06)**

b) Find $L^{-1}\left(\tan^{-1}\left(\frac{2}{s^2}\right)\right)$ **(06)**

c) If R is the relation on the set of integers such that aRb if and only if 2a+3b is divisible by 5. Find the equivalence classes. **(08)**

Q-4

a) Evaluate $\int_{t=0}^{\infty} e^{-3t}\left(\frac{\cos(7t)-\cos(11t)}{t}\right)dt$ **(06)**

b) Find $L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+10)(s^2+2s+17)}\right]$ **(06)**

c) Find the bilinear Transformation which maps the points $2, i, -2$ on to the points $1, i, -1$. Also find image of $|z|=1$ of z-plane to w-plane. **(08)**

Q-5

a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? **(06)**

b) Find the analytic function $f(z)=u+iv$ in terms of z if $u-v=(x-y)(x^2+4xy+y^2)$. **(06)**

c) Solve $\frac{d^3y}{dt^3}-2\frac{d^2y}{dt^2}+5\frac{dy}{dt}=0$ with y(0) = 0, y '(0) =0, y'' (0) = 1. **(08)**

Q-6

a) Prove that $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$ **(06)**

b) Draw the Hasse diagram of D_{105}. **(06)**

c) Find Laplace Transformation of the following

i) $te^{3t}erf\left(5\sqrt{t}\right)$,

ii) $\sin tH(t)+(\cos t-\sin t)H(t-\pi)$ **(08)**
