Duration: 3 hrs **Total Marks: 80**

N.B: (1) Question No. 1 is Compulsory
(2) Attempt any three questions of the remaining five questions
(3) Figures to the right indicate full marks
(4) Make suitable assumptions wherever necessary with proper justifications

1. (a) Using laws of logic prove that **$(p \rightarrow q \wedge \sim q) \vee (r \rightarrow s \wedge \sim s) \rightarrow \sim(p \wedge r)$** is a Tautology **(5)**

   (b) Find number of integers between 1 to 4000 which are: **(5)**
   (i)Divisible by 2 or 3 or 5
   (ii)Divisible by 3 but not by 2 or 5

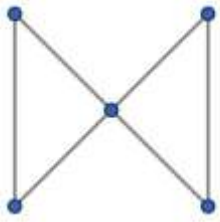
   (c) Find the generating function for the following sequences: **(5)**
   (i) 1,2,3,4…. **(ii)** 2,2,2,2

   (d) Explain the term Partition set. Create the partition based on the following criteria
   Subset 1 = Elements with a remainder of 1 when divisible by 3.
   Subset 2 = Elements with a remainder of 2 when divided by 3. **(5)**
   Subset 3 = Elements divided by 3.

2. (a) Prove that G = {1,2,3,4,5,6,7,8} is an Abelian Group under multiplication modulo 9. **(10)**

   (b) Solve the recurrence relation $a_n - a_{n-1} - 6a_{n-2} = -30$ where $a_0=20$, $a_1=-5$ **(10)**

3. (a) Define Euler path, Euler circuit, Hamiltonian Path, Hamiltonian Circuit. Determine if the following graph has Euler Path, Euler Circuit, Hamiltonian Path, Hamiltonian Circuit and state the path/circuit. **(8)**



   (b)Draw the Hasse Diagram for D_{60} and D_{105}. Are a Lattice? Justify. Find complement of each element. **(8)**

   (c) You pull a number of socks from the drawer without looking. What is the minimum number of pulled socks required to guarantee a pair of the same colour? **(4)**

4. (a) Let R and S be equivalence relations on set A = {m, n, o, p, q} given by R = {(m,m), (m,n), (n,m), (n,n), (o,o), (o,p), (p,o), (p,p), (q,q)} and S = {(m,m), (n,n), (o,o), (p,p), (p,q), (q,p), (q,q)}.
   Find the smallest equivalence relation containing both R and S. **(8)**

(b) Let

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

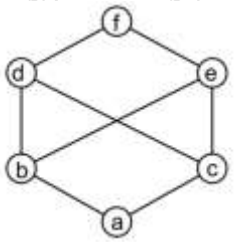
be a parity check matrix. Determine the (2,5) group code function. **(8)**

(c) A bag contains fruits i.e 9 Mangoes and 6 Apples. In how many ways can you draw 5 fruits from the bag such that (i) they it can be any fruit (ii) All are of the same type **(4)**

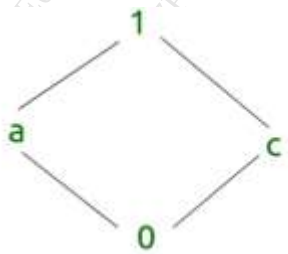
5. (a) Consider functions f, g and h defined as follows: **(8)**
   f: R → R, f(x) = 6x
   g: R → R, g(x) = 3x + 2
   h: R→ R, h(x) = 2x + 6
   Find **g ∘ f, h ∘ f, f ∘ g ∘ h, g ∘ f ∘ h**

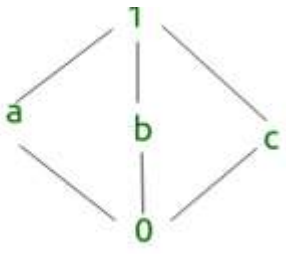
   (b) Consider the following (3,6) group encoding function e: B^3 -> B^6 defined by **(8)**
   e(000) = 000000 e(001) = 000110 e(010) = 010010
   e(011) = 010100 e(100) = 100101 e(101) = 100011
   e(110) = 110111 e(111) = 110001
   Decode the following words relative to a maximum likelihood decoding function.

   (c) Is the following structure a Lattice? Justify your answer **(4)**

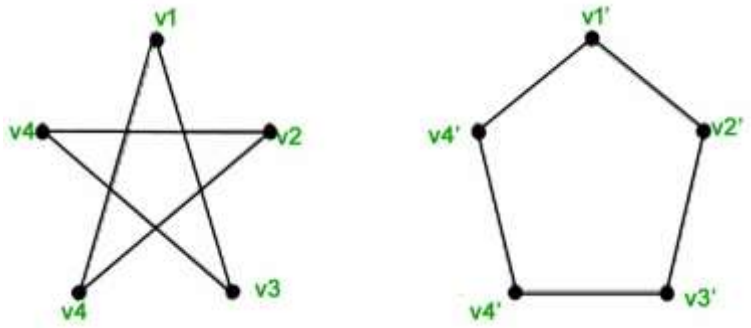


6. (a) What is a Lattice? Explain with example. Identify if the following lattices are distributive or not. Justify your answer. **(8)**





(b) Define Isomorphism in graphs. Find out if the two graphs given below are isomorphic or not. Justify your answer. **(8)**



(c) What are different types of Normal Forms. Explain with examples for each. simplify this logical expression. **p→(q→(p∧q))∧((p∧q)→r)** **(4)**

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