Duration: 3hrs **[Max Marks:80]**

N.B. : (1) Question No 1 is Compulsory.
(2) Attempt any three questions out of the remaining five.
(3) All questions carry equal marks.
(4) Assume suitable data, statistical tables if required and state it clearly.

1

a Use Cayley - Hamilton theorem to find $2A^4 - 5A^3 - 7A + 6I$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ [5]

b If $\bar{F} = (ax + 3y + 4z)i + (x - 2y + 3z)j + (3x + 2y - z)k$ is solenoidal, find the value of a. [5]

c A discrete random variable has the probability distribution given below [5]

X	-2	-1	0	1	2	3
P(X=x)	0.2	k	0.1	2k	0.1	2k

Find k, mean and variance.

d A continuous random variable with p.d.f. $f[x] = k\,x^2(1-x^3),\ 0 \le x \le 1$. Find k [5]
Find mean and variance

2 a Find eigenvalues and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [6]

b Using Green's theorem evaluate $\int (xy + y^2)dx + x^2 dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$ [6]

C Investigate the association between the darkness of eye colour in father and son from the following. [8]

Color of son's eyes	Color of father's eyes	
	Dark	Not Dark
Dark	48	90
Not Dark	80	782

3 a If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ then find A^{100} [6]

b The marks obtained by 1000 students in an examination are found to be normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75. [6]

C By using Big M method solve Minimize $Z = 2x_1 + 3x_2$ Subject to $x_1 + x_2 \geq 5$ [8]
$x_1 + 2x_2 \geq 6$; $x_1\ x_2 \geq 0$

4 a . Individuals are chosen at random from population and their heights are found to 63,63,64,65,66,69,69,70,71,70 inches. Discuss the suggestions that mean height of the population is 65 inches. [6]

b Show that $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalizable. Also find the diagonal form and the transforming matrix [6]

C Solve the following LPP by simplex method [8]

$$Max.Z = 4x_1 + 10x_2$$

Subject to constraint

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

5 a Show that the matrix A is derogatory and find its minimal polynomial A = [6]
$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$

b Samples of two types of electric bulbs were tested for the length of life and following data were obtained [6]

	Type I	Type II
No. of samples	8	7
Mean of sample (in hrs.)	1210	1314
Standard Deviation (in hrs.)	36	42

Test at 5% level of significance whether the difference in the sample mean is significant.

C Evaluate by using Stokes theorem $\int xydx + xy^2\, dy$ where c is the square in xy plane with vertices (1,0),(0,1),(-1,0) and (0,-1) [8]

6 a Use the dual simplex method to solve the following LPP [6]

$$Min.Z = 6x_1 + x_2$$

Subject to the constraints

$$2x_1 + x_2 \geq 3$$

$$x_1 - x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

b In a factory production can be achieved by four different workers on five different types of machines a sample study was made for two fold objectives of examining whether the four differ with respect to mean productivity and whether the mean productivity is the same for five different machine. The researcher involved in this study while analysing the collected data ,reports as follows, [6]

1. Sum of squares of variances between machines =35.2
2. Sum of squares of variances between workers =53.8
3. Sum of squares of variances =174.2

   Construct an ANOVA table for the given information and draw the interference at 5 % level.

C Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1$ into canonical form and hence find rank, index and signature of the matrix [8]