

(Time: 3 hours)

Total Marks: 80

N.B.: (1) Question No. 1 compulsory.

(2) Attempt any Three questions from remaining five questions.

Q1 a) Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -256$ [5]

b) Express the matrix $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as the sum of the symmetric and a skew symmetric matrix. [5]

c) If $p = y^2 + z^2$, $q = z^2 + x^2$, $r = x^2 + y^2$ then evaluate $\frac{\partial(p,q,r)}{\partial(x,y,z)}$. [5]

d) Using Newton-Raphson method for the equation $x^3 - 2x - 5 = 0$, find the root starting with $x_0 = 2$ as initial value with an accuracy of .0001. [5]

Q2 a) Test for consistency and if possible solve

$$x + 2y - z = 2, 3x + 8y + 2z = 10, 4x + 9y - z = 12$$
 [6]

b) Find all the values of $(1 - i\sqrt{3})^{\frac{1}{4}}$.

[6]

c) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$, P.T $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u)$ [8]

Q3 a) Separate into real and imaginary parts $\cos^{-1}\left(\frac{3i}{4}\right)$ [6]

b) Find the Rank of the following matrix by reducing to Normal Form

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix}$$
 [6]

c) Examine the function

$$f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4 \text{ for extreme values.}$$
 [8]

Q4 a) Examine whether the vectors

$$X_1 = [1, 1, 1], X_2 = [2, 3, 8], X_3 = [1, 2, 3] \text{ are linearly independent} \quad [6]$$

b) If $\sin(\alpha + i\beta) = x + iy$, then prove that $[6]$

$$\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1 \quad \text{and} \quad \frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$$

c) If $y = \cos x \cos 2x \cos 3x$ then find n^{th} derivative of y $[8]$

Q5 a) Apply Jacobi's Iterative method to solve the following equations

$$20x + y - 2z = 17, \quad 3x + 20y - z = -18, \quad 2x - 3y + 20z = 25 \quad [8]$$

b) If $v = (1 - 2xy + y^2)^{-\frac{1}{2}}$ then show that $xv_x - yv_y = y^2 v^3$ $[6]$

c) Expand $\log(1 + x + x^2 + x^3)$ up to x^5 $[6]$

Q6 a) Considering only the principle values, prove that the real part of

$$(1 + i\sqrt{3})^{(1+i\sqrt{3})} \text{ is } 2e^{\frac{-\pi}{\sqrt{3}}} \cos\left(\frac{\pi}{3} + \sqrt{3}\log 2\right) \quad [6]$$

b) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, then prove that $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z} = 0$. $[6]$

c) Prove that $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ $[8]$
