(3 Hours) **[Total marks : 80**

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) Find the angle between the surfaces $x \log z + 1 - y^2 = 0, \quad x^2y + z = 2$ at $(1, 1, 1)$. 05

b) Show that the functions $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on $(-1, 1)$. Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that interval. 05

c) Find the Laplace transform of $\int_0^t u^{-1} e^{-u} \sin u \; du$. 05

d) Prove that $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and find $f'(z)$ and $f(z)$ in terms of z. 05

Q.2 a) Obtain half- range sine series of $f(x) = x(\pi - x)$ in $(0, \pi)$ and hence, find the value of $\sum \frac{(-1)^n}{(2n-1)^3}$. 06

b) Prove that $\bar{F} = (y^2 \cos x + z^3)\, i + (2y \sin x - 4)\, j + (3xz^2 + 2)\, k$ is a conservative field. Find the scalar potential for $\bar{F}$. 06

c) Find the inverse Laplace transform of 08

(i) $\frac{s+2}{s^2 - 4s + 13}$

(ii) $\frac{1}{(s-a)(s-b)}$

Q. 3 a) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3 - x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$. 06

b) Find the analytic function $f(z) = u + iv$ if $3u + 2v = y^2 - x^2 + 16xy$. 06

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c) Expand $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ period 2 into a Fourier Series. 08

Q. 4 a) Prove that 06
$\int x^3 \cdot J_0(x)\,dx = x^3 \cdot J_1(x) - 2x^2 \cdot J_2(x)$.

b) Use Stoke's Theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ 06
where $\bar{F} = yz\,i + zx\,j + xy\,k$
and C is the boundary of the circle $x^2 + y^2 + z^2 = 1,\ z = 0$.

c) Solve using Laplace transform $(D^2 - 3D + 2)\,y = 4e^{2t}$ with $y(0) = -3$ and $y'(0) = 5$. 08

Q. 5 a) Prove that $2\,J_0''(x) = J_2(x) - J_0(x)$. 06

b) Use Laplace transform to evaluate 06
$\int_0^\infty e^{-t} \left(\int_0^t u^2 \sin hu \ \cos hu \ du \right) dt$.

c) Obtain complex form of Fourier Series for $f(x) = e^{ax}$ in $(-\pi, \pi)$ where a is not an integer. Hence deduce that when α is a constant other than an integer 08

$$\cos \alpha x = \frac{\sin \pi\alpha}{\pi} \sum \frac{(-1)^n \alpha}{(\alpha^2 - n^2)}\ e^{inx}$$

Q. 6 a) Express the function 06

$$f(x) = \begin{cases} -e^{kx} & for\ \ x < 0 \\ e^{-kx} & for\ x > 0 \end{cases}$$

as Fourier Integral and hence, prove that

$$\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2}\, d\omega = \frac{\pi}{2}\ e^{-kx} \quad if \quad x > 0, k > 0.$$

b) Using Green's theorem evaluate 06

$$\oint_C \left(e^{x^2} - xy\right) dx - (y^2 - ax)\, dy$$

where C is the circle $x^2 + y^2 = a^2$.

c) Under the transformation $w = \frac{z-1}{z+1}$, show that the map of the straight line $y = x$ is a circle and find its center and radius. 08

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