

(3 Hours)

[Total Marks : 80]

Note:-

- 1) Question number 1 is compulsory.
- 2) Attempt any **three** questions from the remaining **five** questions
- 3) **Figures** to the **right** indicate **full** marks.

- Q.1 a) Find the Laplace transform of $\cos t \cos 2t \cos 3t$ 05
- b) Show that the set of functions $\cos nx$, $n = 1, 2, 3, \dots$ is orthogonal over $(0, 2\pi)$ 05
- c) Prove that $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and find $f'(z)$ in terms of z . 05
- d) Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at $(1, 2, 3)$ 05
- Q.2 a) Find the fourier series for $f(x) = x^2$ in $(0, 2\pi)$ 06
- b) Show that the vector $\vec{F} = (x^2 + xy^2) \mathbf{i} + (y^2 + x^2y) \mathbf{j}$ is irrotational and find its scalar potential 06
- c) Prove that the transformation $w = \frac{1}{z+i}$ transforms real axis of z - plane into a circle of w - plane 08
- Q.3 a) Using convolution theorem, find inverse Laplace transform of $\frac{s^2}{(s^2+2^2)^2}$. 06
- b) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$ 06
- c) Find half range cosine series for $f(x) = x(\pi - x)$, $0 < x < \pi$. Hence show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$ 08

Q.4 a) Evaluate by Green's theorem $\int_c (e^{x^2} - xy) dx - (y^2 - ax) dy$ where c is the circle $x^2 + y^2 = a^2$. 06

b) Prove that $2 J_0''(x) = J_2(x) - J_0(x)$. 06

c) i) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$ 08

ii) Find Laplace transform of $t \sqrt{1 + \sin t}$

Q.5 a) Find the orthogonal trajectory of the family of curves $x^3 y - xy^3 = c$. 06

b) Prove that $\int x \cdot J_{2/3}(x^{3/2}) dx = -\frac{2}{3} x^{-1/2} J_{-1/3}(x^{3/2})$. 06

c) Obtain complex form of Fourier Series for $f(x) = e^{2x}$ in $(0, 2)$. 08

Q.6 a) Use Stoke's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = yz \mathbf{i} + zx \mathbf{j} + xy \mathbf{k}$ 06

and C is the boundary of the circle $x^2 + y^2 + z^2 = 1$ and $z = 0$.

b) Find the Fourier integral representation for 06

$$f(x) = e^{ax}, \quad x \leq 0, \quad a > 0$$

$$= e^{-ax}, \quad x \geq 0, \quad a > 0$$

$$\text{Hence show that } \int_0^\infty \frac{\cos wx}{w^2 + a^2} dx = \frac{\pi}{2a} e^{-ax}, \quad x > 0, \quad a > 0$$

c) Solve using Laplace transform $(D^2 + 2D + 5)y = e^{-t} \sin t$, where $y(0) = 0, y'(0) = 1$. 08
