Time 3 Hours **Max. Marks: 80**

Note: (1) Question No. 1 is Compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks.

1. (a) Find the constants a, b, c, d, e if
$f(z)= (ax^4+bx^2y^2+cy^4+dx^2-2y^2) +i (4x^3y-exy^3+4xy)$ is analytic. (5)
(b) Find $L\{e^{-t}\sin 2t\cos 3t\}$. (5)
(c) Use Cayley Hamilton theorem for A= $\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ to find A^3 and A^{-1}. (5)
(d) Obtain the Fourier Series of $f(x) = x^4$, in (-1,1). (5)

2. (a) Find $L^{-1}\left(\frac{s^2}{(s^2+5)(s^2+4)}\right)$ (6)
(b) Find the analytic function f(z)=u + iv where $u + v = e^x (\cos y + \sin y)$. (6)
(c) Find a Fourier series to represent the function
$$f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ \frac{1}{4}\pi x, & 0 < x < \pi \end{cases}$$
Hence, deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ .... (8)

3 (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 1\\ 0 & 0 & 2\end{bmatrix}$ (6)
(b) Find the Laplace transform of $e^{-4t}\int_0^t u\ \sin 3u\ du$ (6)
(c) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, given
$u(0,t) = 0\ , u(4,t) = 0, u(x,0) = x^2(16 - x^2)$
Assume h=1 upto t = 1 sec (8)

4 (a) Find the orthogonal trajectory of the family of curves given by $e^x\cos y - xy = c$ (6)

(b) Find $L^{-1}\left[\frac{(s+3)^2}{(s^2+6s+18)^2}\right]$ using convolution theorem (6)

(c) Show that $A = \begin{pmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{pmatrix}$ is diagonalizable. Determine a transforming matrix and a diagonal matrix. (8)

5 (a) Find half range cosine series for $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ x, & 1 \le x \le 2 \end{cases}$ (6)

(b) By using Laplace transform, evaluate $\int_0^\infty \frac{sin2t+sin3t}{te^t} dt$ (6)

(c) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 \le x \le 1$ subject to the condition
$u(0,t) = 0, u(1,t) = 0, u(x,0)$ =100 (x- x^2), h = 0.25 for one time step. (8)

6 (a) Find $L^{-1}\left[log\frac{(s^2+4)}{(s+2)^2}\right]$ (6)

(b) Find sin A where A = $\begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$ (6)

(c) Find a Fourier series for $f(x)$ in $(0, 2\pi)$ Where

$$f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi \le x < 2\pi \end{cases}$$

Hence, deduce that $\frac{\pi^2}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ .... (8)

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