

Time: 3 hours

Max. Marks: 80

- Note: 1. Q.1 is compulsory
 2. Attempt any **three** out of remaining.
 3. Right indicates full marks.

Q.1

- a) How many friends must you have to guarantee that at least five of them have their birthday in the same month? [5]
 b) Find Laplace transformation of $f(t) = t \sin 3t \cos t$ [5]
 c) Find the constant a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic. [5]
 d) Function f and g are defined as follows $f: R \rightarrow R, g: R \rightarrow R$ $f(x) = 2x + 3, g(x) = 3x + 2$ find $f \circ g, g \circ f$ [5]

Q.2

- a) Find inverse Laplace transformation by using convolution theorem $\frac{s^2}{(s^2+9)(s^2+16)}$. [6]
 b) Prove that $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and find $f'(z)$ in terms of z. [6]
 c) Evaluate $\int_0^\infty e^{-t} \left(\frac{\cos 6t - \cos 4t}{t} \right) dt$. [8]

Q.3

- a) If any 14 integers from 1 to 26 are chosen then show that at least one of them is multiple of another. [6]
 b) Let $A = \{1, 2, 3, 6\}$ and $B = \{1, 2, 3, 6, 7, 14, 21, 42\}$ and R be the relation "is divisible by". Draw Hasse diagram for the two sets. Show that they are poset. [6]
 c) Evaluate [8]
 (i) $L^{-1} \left[\frac{s+2}{s^2+4s+7} \right]$ (ii) $L^{-1} \left[\log \left(\frac{s+2}{s+3} \right) \right]$

Q.4

- a) Find the orthogonal trajectories of the family of curve $2x - x^3 + 3xy^2 = a$ [6]
 b) In how many different ways can 6 men and 6 women be seated in a row if men and women must occupy alternate seats? [6]
 c) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 1), (2, 3), (3, 4)\}$. Find the transitive closure by Warshall's algorithm. [8]

Q.5

- a) 75 children went to an amusement park where they can ride on a merry-go-round, roller coaster and ferris wheel. It is known that 20 of them have taken all three rides, and 55 of them have taken at least two of three rides. Each ride cost Rs 50 and the total receipt of the park was Rs. 7000. Determine the number of children who didn't try any of the rides. [6]
 b) Three factories A, B, C produce 30%, 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by factory A. [6]
 c) Find bilinear transformation which maps the points $z = 1, i, -1$ onto the points $0, 1, \infty$. [8]

Q.6

- a) Find the Laplace transformation of $\int_0^t e^{-2u} \cos^2 2u \, du$ [6]
- b) Solve the following equation by using Laplace transform $3 \frac{dy}{dt} + 2y = e^{3t}$, $y=1$ at $t=0$ [6]
- c) Each coefficient of equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots. [8]
