Q. P. Code: 24361

(3 Hours) **[Total Marks: 80]**

N.B. : 1) Question No. **1** is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) If λ is eigen value of A and X is corresponding eigen vector of λ then show that λ^n is eigen value of A^n and corresponding eigen vector is X $(n>0)$. (5)

(b) Evaluate $\int_C \frac{z^2-2z+4}{z^2-1}\,dz$, where C is $|z-1|=1$. (5)

(c) Find the extremals of $\int_{x_1}^{x_2}(1+x^2y')y'\,dx$. (5)

(d) Find a unit vector orthogonal to both $u=(-3,2,1)$ and $v=(3,1,5)$. (5)

Q.2 (a) Find eigen values and eigen vectors of A^2+2I where $A=\begin{bmatrix}8 & -8 & -2\\ 4 & -3 & -2\\ 3 & -4 & 1\end{bmatrix}$. (6)

(b) Find the extremals of $\int_{x_1}^{x_2}\left[(y'')^2-y^2\right]dx$. (6)

(c) Obtain Laurent's series expansion of $f(z)=\frac{4z+3}{z^2-z-6}$ at $z=1$. (8)

Q.3 (a) Using Rayleigh-Ritz method find solution for the extremal of the functional $\int_0^1\left[(y')^2-2y-2xy\right]dx$ with $y(0)=2$ and $y(1)=1$. (6)

(b) Evaluate $\int_0^\infty \frac{1}{(x^2+1)(x^2+9)}\,dx$. (6)

(c) Show that matrix $A=\begin{bmatrix}-9 & 4 & 4\\ -8 & 3 & 4\\ -16 & 8 & 7\end{bmatrix}$ diagonalizable. Also find diagonal and transforming matrix. (8)

[Turnover]

Q.4

a) Verify Cayley Hamilton Theorem for $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. Also find A^{-1}. (6)

(b) Using Cauchy's Residue Theorem evaluate $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$. (6)

(c) Show that the extremal of isoperimetric problem $I=\int_{x_1}^{x_2} (y')^2\, dx$ subject to the condition $\int_{x_1}^{x_2} y\, dx = k$ is a parabola. (8)

Q.5 (a) Find 5^A where $A=\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. (6)

(b) Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process where $S=\{(1,1,1),(-1,1,0),(1,2,1)\}$ (6)

(c) Reduce the following quadratic form into canonical form and hence find its rank, index, signature and value class (8)
$Q=5x_1^2+26x_2^2+10x_3^2+6x_1x_2+4x_2x_3+14x_3x_1$.

Q.6 (a) State and prove Cauchy-Schwartz inequality. Hence show that for real values of a, b, θ $(a\cos\theta+b\sin\theta)^2 \le a^2+b^2$. (6)

(b) Show that any plane through origin is a subspace of R^3. (6)

(c) Find the singular value decomposition of $A=\begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$. (8)
