Q.P. Code : 24851

[Time: Three Hours] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B: 1. Question No.1 is compulsory.
2. Answer any three from the remaining.
3. Figures to the right indicate marks.

Q.1. a. Separate into real part and imaginary of Cos$^{-1}\left(\frac{3i}{4}\right)$ **03**

b. Show that the matrix A is unitary where A = $\begin{bmatrix} \propto + i\gamma & \beta + i\delta \\ \beta + i\delta & \propto + i\gamma \end{bmatrix}$ is unitary if $\propto^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ **03**

c. If z = tan $(y + ax)$+ $(y - ax)^{3/2}$ then show that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$ **03**

d. $If\ x = uv \quad y = \frac{u}{v}$ Prove that JJ^I= 1 **03**

e. Find the nth derivative of $\frac{x^3}{(x+1)(x-2)}$ **04**

f. Using the matrix A = $\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$ decode the message matrix C = $\begin{bmatrix} 4 & 11 & 12-2 \\ -4 & 4 & 9-2 \end{bmatrix}$ **04**

Q.2. a. If $sin^4\theta\ cos^3\theta$ = a cosθ + b cos 3θ+ C cos 5θ+ d cos 7θ then find a, b, c, d. **06**

b. Using Newton Raphson method Solve $3x - Cosx - 1 = 0$ Correct to 3 decimal places. **06**

c. Find the stationary points of the function x^3+$3xy^2$$-3x^2$$-3y^2 + 4$ & also find maximum and minimum values of the function. **08**

Q.3. a. Show that **06**

x cosec x = 1+ $\frac{x^2}{6}$ +$\frac{7}{360}$ x^4+ -------

b. Reduce matrix to PAQ normal form and find 2 non Singular matrices P & Q **06**

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

c. If y= cos (m sin^{-1}x) Prove that $(1 - x^2)y_{n+2}$ – $(2n + 1)xy_{n+1}$ + $(m^2 - n^2)y_n$ =0 **08**

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Q.4. a. State and prove Euler's theorem for three Variables. **06**

b. Show that all the roots of $(x+1)^6 + (x-1)^6 = 0$ are given by $-i\cot\frac{(2k+1)\pi}{12}$ where k= 0,1,2,3,4,5 **06**

c. Show that the equations **08**

$$-2x + y + z = a$$
$$x - 2y + z = b$$
$$x + y - 2z = c$$

have no solutions unless a + b + c = 0 in which case they have infinitely many solutions. Find these Solutions when $a = 1 \quad b = 1 \quad c = -2$

Q.5. a. If $z = f(x,y) \quad x = r\cos\theta$ **06**

$y = r\sin\theta$ Prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

b. If $\cos hx = \sec\theta$ Prove that **06**

i) $x = \log(\sec\theta + \tan\theta)$

ii) $\theta = \frac{\pi}{2} - 2\tan^{-1}(e^{-x})$

c. Solve by Gauss Jacobi Iteration method **08**

$$5x - y + z = 10$$
$$2x + 4y = 12$$
$$x + y + 5z = -1$$

Q.6. a. Prove that **06**

$$\cos^{-1}[\tan h(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - - - - -\right)$$

b. If $y = e^{2x}\, Sin\,\frac{x}{2}\, cos\,\frac{x}{2} \sin 3x$ Find y_n **06**

c. (i) Evaluate $\lim_{x \to 0} (Cot\, x)^{\sin x}$ **04**

(ii) Prove that $\log\left[\frac{\sin(x+iy)}{\sin(x-iy)}\right] = 2i\tan^{-1}(\cot x \tan hy)$ **04**

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