University of Mumbai

Program: First Year (All Branches) Engineering

Curriculum Scheme: Rev2019

Examination: FE Semester II

Course Code: _FEC201 Course Name: Engineering Mathematics II

Time: 2 hour 30 minutes Max. Marks: 80

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Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry TWO marks (20 Marks)**
1.	Particular Integral of DE $(D^3+3D^2-4)y=e^x$ is
Option A:	$xe^x/9$
Option B:	$xe^x/2$
Option C:	$-xe^x/9$
Option D:	$xe^x/6$
2.	The solution of the differential equation $\left(x+\frac{e^x}{y}\right)dx-\frac{e^x}{y^2}dy=0$ is
Option A:	$\frac{x^2}{2}+\frac{e^x}{y}=c$
Option B:	$\frac{x^2}{3}+\frac{e^x}{y}=c$
Option C:	$\frac{x^3}{2}+\frac{e^x}{y}=c$
Option D:	$\frac{x^2}{2}+\frac{xe^x}{y}=c$
3.	The value of $\int_0^{\infty} x^5 e^{-x^2}dx$ is
Option A:	0
Option B:	1
Option C:	1/2

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Option D:	π
4.	The value of I = $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy\, dx$ is
Option A:	$\frac{3}{35}$
Option B:	$\frac{3}{15}$
Option C:	$\frac{1}{35}$
Option D:	$\frac{3}{5}$
5.	The value of $\int_0^{\pi/2} \int_0^{a\cos\theta} \int_0^{r/\cos\theta} dz\; dr\; d\theta$ is
Option A:	0
Option B:	$\frac{a^2}{8}$
Option C:	$\frac{a^3}{3}$
Option D:	$\frac{a^2}{2}$
6.	The Integrating Factor of DE $(x^2 e^x - my)dx + mx\, dy = 0$ is given by
Option A:	$\frac{1}{y^2}$
Option B:	$\frac{1}{x^2}$
Option C:	$-\frac{1}{y^2}$
Option D:	$-\frac{1}{x^2}$
7.	Find the complementary function of $\frac{d^4y}{dx^4} + \frac{5\, d^2y}{dx^2} + 4 = x \sin x$
Option A:	$y = C_1 \cos x + C_2 \sin x + C_3 \cos 3x + C_4 \sin 3x$
Option B:	$y = C_1 \cos x + C_2 \sin x + C_3 \cos 2x + C_4 \sin 2x$

Option C:	$y = C_1 \cos xi + C_2 \sin xi + C_3 \cos 2xi + C_4 \sin 2xi$
Option D:	$y = (C_1 + C_2x) \cos x + (C_3 + C_4x) \sin 2x$
8.	Changing the order of integration in double integral $\int_0^2 \int_0^{4-x^2} f(x,y)dydx$ leads to $\int_a^b \int_c^d f(x,y)dxdy$ then value of 'd' is
Option A:	$4 - y$
Option B:	$2 - y$
Option C:	$\sqrt{4-y}$
Option D:	0
9.	The length of the straight line $y = 2x + 5$ $from$ $x = 1$ to $x = 3$ is given by
Option A:	$\sqrt{5}$ units
Option B:	$3\sqrt{5}$ units
Option C:	$4\sqrt{5}$ units
Option D:	$2\sqrt{5}$ units
10.	Evaluate: $\int_0^{log2} \int_0^x \int_0^{x-y} e^{x+y+z}\, dzdydx$
Option A:	2iog2$-\frac{5}{4}$
Option B:	2log2$+\frac{5}{8}$
Option C:	log2$-\frac{5}{4}$
Option D:	2log2$-\frac{1}{4}$

Q2.	**Solve any Four out of Six (5 marks each)** **(20 Marks)**
A	Solve the DE $(2xy\cos x^2 - 2xy + 1)\,dx + (\sin x^2 - x^2)\,dy = 0$
B	Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$
C	Prove that $\int_0^\infty \frac{1-\cos ax}{x} e^{-x}\,dx = \frac{1}{2}\log(1 + a^2)$, assuming the validity of differentiation under the integral sign.
D	Change the order of integration and evaluate $\int_0^1 \int_{-\sqrt{y}}^{-y^2} xy\,dx\,dy$
E	Evaluate $\int\int\int z\,dz\,dy\,dx$, over the tetrahedron bounded by $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$
F	Find the length of the cardioid r= a(1-cosθ) lying outside the circle r = acosθ.

Q3.	**Solve any Four out of Six** **5 marks each** **(20 Marks)**
A	Solve $\tan y\frac{dy}{dx} + \tan x = \cos y \cdot \cos^3 x$
B	Solve the DE $(D^2 - 2D + 1)y = x^2 e^{3x}$ $, where\ D \equiv \frac{d}{dx}$
C	Evaluate $\int_0^\infty x^2\, 5^{-4x^2} dx$
D	Evaluate the integral $I = \int\int xy\,(x + y)\,dx\,dy$ over the region bounded by the curves $y = x^2$ & $y = x$.
E	Evaluate $\iiint dxdydz$ over the solid of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane z = 4
F	Find the area common to $r = a(1 + \cos\theta)$ & $r = a(1 - \cos\theta)$

Q4.	**Solve any Four out of Six** **5 marks each** **(20 Marks)**
A	Solve $xy - \frac{dy}{dx} = y^3 e^{-x^2}$
B	Solve $\frac{d^2y}{dx^2} - y = x\sin x + \cos x$
C	Change the order of integration and evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$
D	Evaluate $\int_0^\infty \frac{x^2}{(1+x^6)^{5/2}}\,dx$
E	Change to polar co-ordinates and evaluate $\int_0^1 \int_0^x x + y\ dydx$

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F	Solve: $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$, using method of variation of parameters

Q. No. 7 (MCQ Section)

Before Correction

Find the complementary function of $\frac{d^4y}{dx^4} + \frac{5\, d^2y}{dx^2} + 4 = x \sin x$

After Correction

Find the complementary function of $\frac{d^4y}{dx^4} + \frac{5\, d^2y}{dx^2} + 4y = x \sin x$

Q1 mcq

1) $\frac{xe^x}{9}$ $\boxed{A}$ 2) $\frac{x^2}{2} + \frac{xe^x}{y} = c$ $\boxed{A}$

3) $\boxed{B}$ 1 4) $\frac{3}{35}$ $\boxed{B}$

5) $\frac{a^2}{2}$ $\boxed{D}$ 6) $\frac{1}{x^2}$ $\boxed{B}$

7) $y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$ $\boxed{B}$

8) $\sqrt{4-y}$ $\boxed{C}$

9) $2\sqrt{5}$ $\boxed{D}$ 10) $\boxed{A}$ $2\log 2 - \frac{5}{4}$

Q2. A)

$$(2xy \cos x^2 - 2xy + 1)\,dx + (\sin x^2 - x^2)\,dy = 0$$

$M = 2xy \cos x^2 - 2xy + 1$, $N = \sin x^2 - x^2$

$\frac{\partial M}{\partial y} = 2x \cos x^2 - 2x$ $\frac{\partial N}{\partial x} = 2x \cos x^2 - 2x$

eq^n is exact

G.S is

$$\int_{y\ \text{constant}} M\,dx + \int_{\text{no } x} N\,dy = c$$

$$\therefore \int (2x\cos x^2 \cdot y - 2xy + 1)\,dx + \int 0\,dy = c$$

$$\therefore y\cdot \sin x^2 - x^2 y + x = c$$

B) $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$$

$$(D^2 + 3D + 2)y = \sin e^x$$

AE is $$D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2, -1$$

$$y_c = c_1 e^{-2x} + c_2 e^{-x}$$

$$y_p = \frac{1}{(D+2)(D+1)} \cdot \sin e^x$$

$$= \left(\frac{1}{D+1} - \frac{1}{D+2}\right) \sin e^x$$

$$= \frac{1}{D+1}\sin e^x - \frac{1}{D+2}\sin e^x$$

$$= e^{-x}\int e^x \cdot \sin e^x\,dx - e^{-2x}\int e^{2x}\cdot \sin e^x\,dx$$

$$e^x = t \quad e^x\,dx = dt$$

$$= e^{-x}\int \sin t\,dt - e^{-2x}\int t\cdot \sin t\,dt$$

$$= e^{-x}(-\cos t) - e^{-2x}(-t\cos t + \sin t)$$

$$= e^{-x}(-\cos e^x) - e^{-2x}(-e^x\cdot \cos e^x + \sin e^x)$$

$$y_p = -e^{-x}\cdot\cos e^{x} + e^{-x}\cos e^{x} - e^{-2x}\cdot\sin e^{x}$$ (3)

$$= -e^{-2x}\cdot\sin e^{x}$$

$$y = y_c + y_p$$

$$= c_1 e^{-2x} + c_2 e^{-x} - e^{-2x}\cdot\sin e^{x}$$

c) $$I(a) = \int_0^{\infty} \frac{1-\cos ax}{x}\, e^{-x}\cdot dx \quad \text{—— (1)}$$

Apply DUIS

$$\frac{dI}{da} = \int_0^{\infty} \frac{+\sin ax}{x}\cdot x\cdot e^{-x}\cdot dx$$

$$= +\int_0^{\infty} e^{-x}\cdot\sin ax\cdot dx$$

$$= +\left[\frac{e^{-x}}{1+a^2}(-1\sin ax - a\cos ax)\right]_0^{\infty}$$

$$= +\left[0 - \frac{1}{1+a^2}(-a)\right]$$

$$\frac{dI}{da} = \frac{+a}{1+a^2} \quad \text{—— (ii)}$$

on integrating

$$I = +\frac{1}{2}\log(1+a^2) + c \quad \text{—— (iii)}$$

put $a=0$ in (1) & (iii)

$$I(1) = 0 \qquad I(1) = \frac{1}{2}\log 1 + c = 0 + c$$

$$c = 0$$

$$I = \frac{1}{2}\log(1+a^2$$

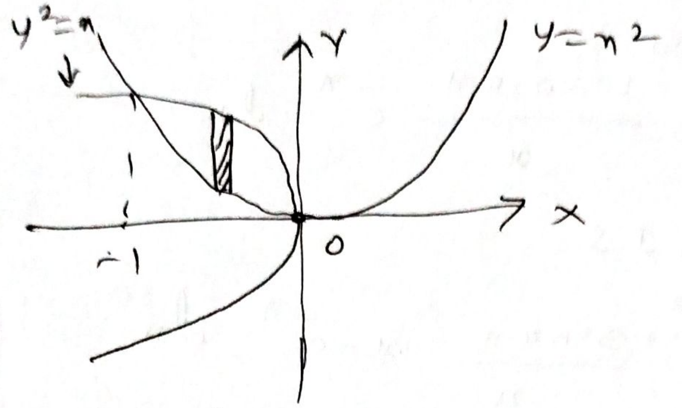
(4)

D> $I = \int_0^1 \int_{-\sqrt{y}}^{-y^2} xy\,dx\,dy$

limits are

$x = -\sqrt{y}$ to $x = -y^2$

$x^2 = y$ & $x^2 = -y^2$ ∴ $y^2 = -x$



$y = x^2$ & $y^2 = -x$

$y = y^4$

$y(y^3 - 1) = 0$

$y = 0,\ y = 1$

$x = 0,\ x = -1$

New limits are

$y = x^2$ to $y = \sqrt{-x}$

$x = 0$ to $x = -1$

$$I = \int_0^{-1} \int_{x^2}^{\sqrt{-x}} y\,dy\,x\,dx$$

$$= \int_0^{-1} \left[\frac{y^2}{2}\right]_{x^2}^{\sqrt{-x}} \cdot x\,dx$$

$$= \frac{1}{2}\int_0^{-1} \left[-\frac{x}{\cancel{2}} - \frac{x^4}{}\right) x\,dx = \frac{1}{2}\int_0^{-1} (-x^2 - x^5)\,dx$$

$$= \cancel{\frac{1}{2}\left[-\frac{x^2}{2} - \frac{x^5}{5}\right]_0^{-1}}$$

$$= \frac{1}{2}\left[\frac{-x^3}{3} - \frac{x^6}{6}\right]_0^{-1}$$

$$= \frac{1}{2}\left[\frac{+1}{3} - \frac{1}{6}\right] = \frac{1}{2}\,\frac{2-1}{6} = \frac{1}{12}$$

E) $I = \iiint z \, dz \, dy \, dx$ $\quad x=0, y=0, z=0$ ⑤

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

put

$x = au \quad y = bv \quad z = cw$

$dx = a\,du \quad dy = b\,dv \quad dz = c\,dw$

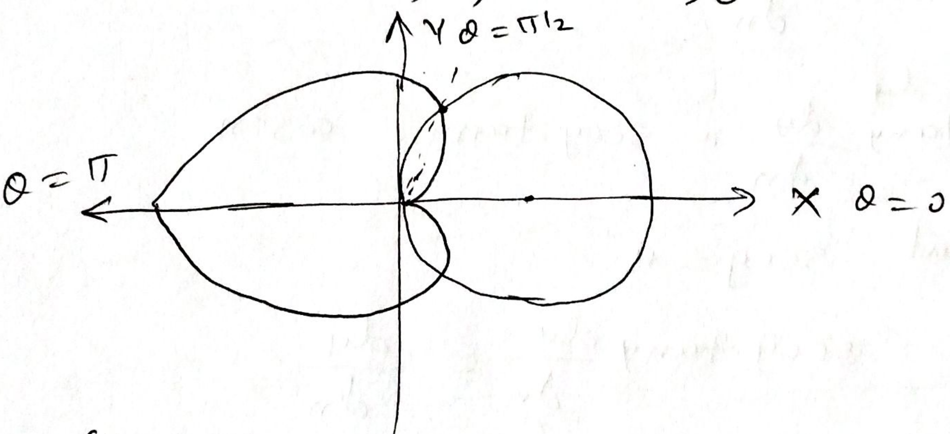
$$= \iiint cw \; a\,du \; b\,dv \; c\,dw$$

$$= abc^2 \iiint x^{1-1} y^{1-1} z^{2-1} \, du\,dv\,dw$$

$$= abc^2 \frac{\Gamma 1 \, \Gamma 1 \, \Gamma 2}{\Gamma 5}$$

$$= abc^2 \cdot \frac{1}{4!} = \frac{abc^2}{24}$$

F) $r = a(1-\cos\theta)$, $r = a\cos\theta$



$r = a(1-\cos\theta)$, $r = a\cos\theta$

$a - a\cos\theta = a\cos\theta$

$2a\cos\theta = a$

$\cos\theta = 1/2$

$\theta = \pi/3$

$r = a(1-\cos\theta)$

$$\frac{dr}{d\theta} = a(\sin\theta)$$

$\theta \to \pi/3$ to π

$$S = 2 \int_{\pi/3}^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

$$= 2\int_{\pi/3}^{\pi} \sqrt{a^2 - 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta}\, d\theta$$ ⑥

$$= 2 \int_{\pi/3}^{\pi} a \cdot \sqrt{2}\sqrt{1-\cos\theta}\, d\theta$$

$$= 2\sqrt{2}\, a \int_{\pi/3}^{\pi} \sqrt{2} \sqrt{\sin^2\theta/2}\, d\theta$$

$$= 4a \int_{\pi/3}^{\pi} \sin\theta/2 \, d\theta$$

$$= 4a \left[- \frac{\cos\theta/2}{1/2} \right]_{\pi/3}^{\pi}$$

$$= 8a \left[0 + \frac{\sqrt{3}}{2} \right] = 4a\sqrt{3}$$

Q3. A)

$$\tan y \frac{dy}{dx} + \tan x = \cos y \cdot \cos^3 x$$

$$\sec y \tan y \cdot \frac{dy}{dx} + \sec y \cdot \tan x = \cos^3 x$$

put $\sec y = u$

$$\sec y \cdot \tan y \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} + u \tan x = \cos^3 x$$

$$If = e^{\int \tan x\, dx} = \sec x$$

Gs is

$$u \times If = \int (If) \times Q\, dx$$

$$u \times \sec x = \int \sec x \times \cos^3 x \, dx$$

$$= \int \cos^2 x\, dx$$

$u \times \sec x = \int \frac{1+\cos 2x}{2}\, dx$ ⑦

$\sec x \cdot \sec y = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$

B) $(D^2 - 2D + 1)y = x^2 e^{3x}$

AE is $(D-1)^2 = 0$

$D = 1, 1$

$y_c = (c_1 + c_2 x)e^x$

$$y_p = \frac{1}{(D-1)^2} x^2 e^{3x}$$

$$= e^{3x} \cdot \frac{1}{(D+3-1)^2} x^2$$

$$= e^{3x} \frac{1}{D^2 + 4D + 4} x^2$$

$$= \frac{e^{3x}}{4} \cdot \left(1 + \frac{D^2 + 4D}{4}\right)^{-1} x^2$$

$$= \frac{e^{3x}}{4}\left[1 - \frac{D^2 + 4D}{4} + \frac{(D^2 + 4D)^2}{16}\right] x^2$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{(D^2 + 4D)x^2}{4} + \frac{(D^4 + 8D^3 + 16D^2)x^2}{16}\right]$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{2 + 8x}{4} + \frac{0 + 0 + 32}{16}\right]$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{2}{4} - 2x + 2\right]$$

$$= \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{2}\right)$$

$y = y_c + y_p$

8

c) $I = \int_0^\infty x^2 \, 5^{-4x^2} dx$

$5 = e^m \therefore m = \log 5$

$= \int_0^\infty x^2 \, e^{-4mx^2}$

$4mx^2 = t$

$x = \frac{1}{2\sqrt{m}} t^{1/2} \quad \therefore x^2 = \frac{1}{4m} t$

$dx = \frac{1}{2\sqrt{m}} \frac{1}{2} t^{-1/2} dt$

$= \int_0^\infty \frac{1}{4m} t \, e^{-t} \cdot \frac{1}{2\sqrt{m}} \frac{1}{2} t^{-1/2} dt$

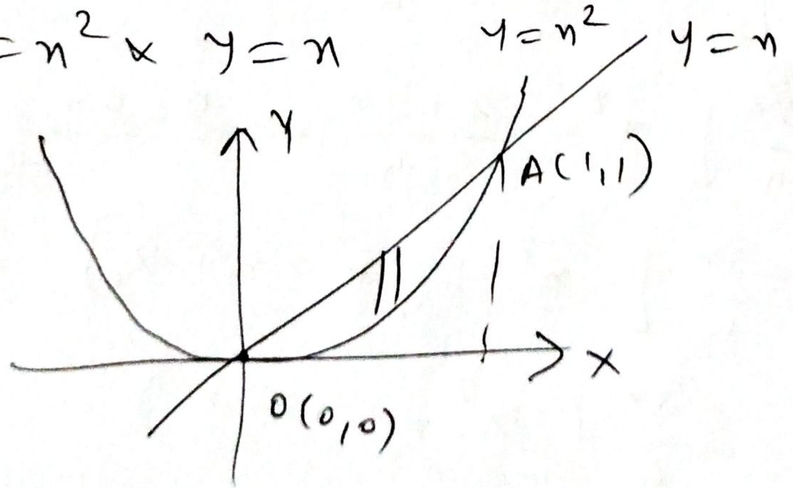
$= \frac{1}{16 m^{3/2}} \int_0^\infty e^{-t} \cdot t^{1/2} dt$

$= \frac{1}{16 m^{3/2}} \Gamma\left(3/2\right) = \frac{\frac{1}{2}\Gamma\left(1/2\right)}{16(\log 5)^{3/2}}$

$= \frac{\sqrt{\pi}}{32(\log 5)^{3/2}}$

D) $I = \iint xy(x+y) \, dx \, dy$

$y = x^2$ & $y = x$



limits are $y = x^2$ to $y = x$

$x = 0$ to $x = 1$

$$I = \int_0^1 \int_{x^2}^{x} (x^2 y + x y^2)\, dx\, dy$$

$$= \int_0^1 \left[\frac{x^2 y^2}{2} + \frac{x y^3}{3} \right]_{x^2}^{x} dx$$

$$= \int_0^1 \left[\frac{x^4}{2} + \frac{x^4}{3} - \frac{x^6}{2} - \frac{x^7}{3} \right] dx$$

$$= \left[\frac{x^5}{10} + \frac{x^5}{15} - \frac{x^7}{14} - \frac{x^8}{24} \right]_0^1$$

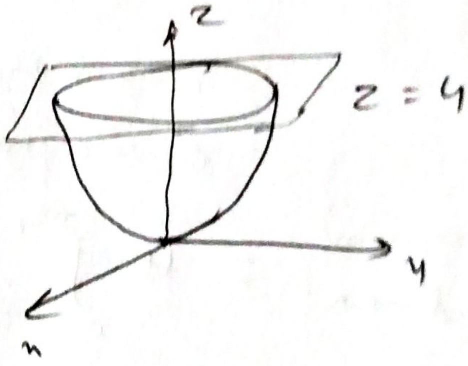
$$= \frac{1}{10} + \frac{1}{15} - \frac{1}{14} - \frac{1}{24}$$

$$= \frac{3}{56}$$

E) $I = \iiint dx\, dy\, dz$ $\qquad x^2 + y^2 = 4z$

$z = 4$

Problem convert to cylindrical polar



$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$dx\, dy\, dz = r\, dr\, d\theta\, dz$

$r \to 0$ to 4 $\quad \theta \to 0$ to 2π, $\quad z \to \frac{r^2}{4}$ to 4

$$I = \int_0^{2\pi} \int_0^4 \int_{\frac{r^2}{4}}^{4} dz\, \cancel{dr}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^4 [z]_{\frac{r^2}{4}}^{4}\, r\, dr\, d\theta$$

$= \int_0^{2\pi} \int_0^4 \left[4 - \frac{r^2}{4}\right] r\,dr\,d\theta$

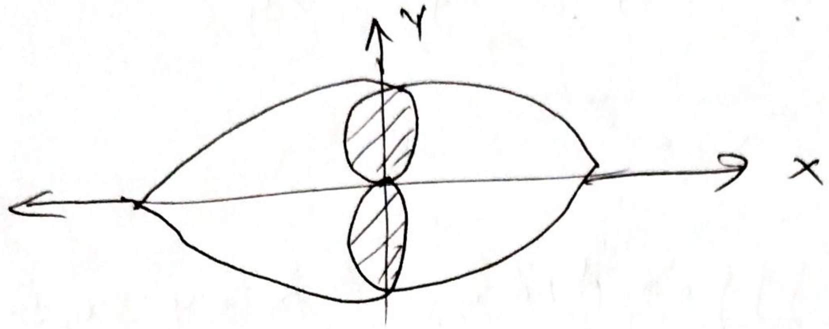
$= \int_0^{2\pi} \int_0^4 \left[4r - \frac{r^3}{4}\right] d\theta$

$= \int_0^{2\pi} \left[\frac{4r^2}{2} - \frac{r^4}{16}\right]_0^4 d\theta$

$= [\theta]_0^{2\pi} \left[\frac{64}{2} - \frac{256}{16}\right]$

$= 2\pi \cdot (32 - 16) = 32\pi$

F)



$\text{Area} = \iint r\,dr\,d\theta$

$r \to 0 \text{ to } a(1-\cos\theta)$

$\theta \to 0 \text{ to } \pi/2$

Required Area $= 4 \int_0^{\pi/2} \int_0^{a(1-\cos\theta)} r\,dr\,d\theta$

$= 4 \int_0^{\pi/2} \left[\frac{r^2}{2}\right]_0^{a(1-\cos\theta)} d\theta$

$= \frac{4}{2} \int_0^{\pi/2} a^2 (1-\cos\theta)^2 \,d\theta$

$= 2a^2 \int_0^{\pi/2} \left(1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta$

(11)

$$= \frac{4a^2}{2} \int_0^{\pi/2} \left(\frac{3}{2} - 2\cos\theta + \frac{\cos 2\theta}{2} \right) d\theta$$

$$= \frac{4a^2}{2} \left[\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4} \right]_0^{\pi/2}$$

$$= \frac{4a^2}{2} \left[\frac{3\pi}{4} - 2 + 0 - 0 \right]$$

$$= 2a^2 \left(\frac{3\pi - 8}{4} \right) = 2a^2 (3\pi - 8)$$

Q4. A) $xy - \frac{dy}{dx} = y^3 e^{-x^2}$

$$\frac{dy}{dx} = xy - y^3 e^{-x^2}$$

$$y^{-3} \frac{dy}{dx} - xy^{-2} = -e^{-x^2}$$

put $y^{-2} = u$

$$-2y^{-3} \frac{dy}{dx} = \frac{du}{dx}$$

$$y^{-3} \frac{dy}{dx} = -\frac{1}{2} \frac{du}{dx}$$

$$-\frac{1}{2} \frac{du}{dx} - ux = -e^{-x^2}$$

$$\frac{du}{dx} + 2ux = 2e^{-x^2}$$

$$IF = e^{x^2}$$

G.S.

$$u \times e^{x^2} = \int 2e^{-x^2} \cdot e^{x^2} dx$$

$$\frac{e^{x^2}}{y} = 2x + C$$

B) $\frac{d^2y}{dx^2} - y = x\sin x + \cos x$

$(D^2-1)y = x\sin x + \cos x$

AE $D^2 - 1 = 0$

$D^2 = 1 \quad D = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$y_p = \frac{1}{D^2-1}(x\sin x + \cos x)$

$= \frac{1}{D^2-1} x\sin x + \frac{1}{D^2-1}\cos x$

$= \left[x - \frac{2D}{D^2-1}\right]\frac{1}{D^2-1}\sin x + \frac{1}{-1-1}\cos x$

$= \left[x - \frac{2D}{-2}\right]\frac{1}{-2}\sin x - \frac{1}{2}\cos x$

$= [x + D]\frac{-1}{2}\sin x - \frac{1}{2}\cos x$

$y_p = -\frac{1}{2}(x\sin x + \cos x) - \frac{1}{2}\cos x$

$y = y_c + y_p$

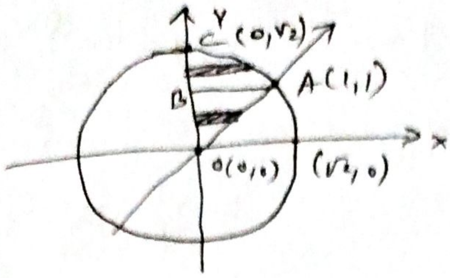
c) $I = \int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dx\, dy$

limits are $y = x$ to $y = \sqrt{2-x^2}$

$x = 0$ to $x = 1$

$y^2 = 2 - x^2$

$x^2 + y^2 = 2$



New limits are

In region OAB $x=0$ to $x=y$

$y=0$ to $y=1$

In region BACB $x=0$ to $x=\sqrt{2-y^2}$

$y=1$ to $y=\sqrt{2}$

$$I = \int_0^1\int_0^y \frac{x}{\sqrt{x^2+y^2}}\,dx\cdot dy + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx$$

$$= \int_0^1 \left[\sqrt{x^2+y^2}\right]_0^y dy + \int_1^{\sqrt{2}} \left[\sqrt{x^2+y^2}\right]_0^{\sqrt{2-y^2}}$$

$$= \int_0^1 (\sqrt{2}y - y)\,dy + \int_1^{\sqrt{2}} (\sqrt{2}-y)\,dy$$

$$= \left(\sqrt{2}\cdot\frac{y^2}{2} - \frac{y^2}{2}\right)_0^1 + \left(\sqrt{2}y - \frac{y^2}{2}\right)_1^{\sqrt{2}}$$

$$= \frac{1}{2}\sqrt{2} - \frac{1}{2} + 2 - 1 - \sqrt{2} + \frac{1}{2}$$

$$= 1 - \frac{1}{\sqrt{2}}$$

2) $$I = \int_0^\infty \frac{x^2}{(1+x^6)^{5/2}}\,dx$$

Put $x^6 = t$ $x = t^{1/6}$

$$dx = \frac{1}{6} t^{-5/6}\,dt$$

$$= \int_0^\infty \frac{t^{2/6}}{(1+t)^{5/2}} \cdot \frac{1}{6} t^{-5/6}\cdot dt$$

$$= \frac{1}{6}\int_0^\infty \frac{t^{-1/2}}{(1+t)^{5/2}}\,dt$$

$$= \frac{1}{6}\int_0^\infty \frac{t^{1/2-1}}{(1+t)^{\frac{1}{2}+4/2}}\,dt = \frac{1}{6}B\left(1/2, 4/2\right)$$

$$= \frac{1}{6}B\left(\frac{1}{2}, 2\right)$$

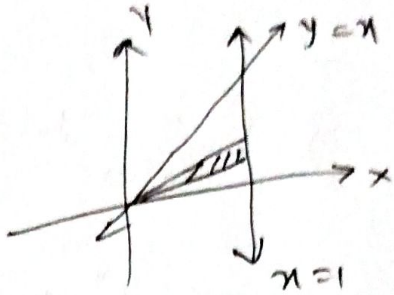
$$\frac{d^2}{d} = \frac{1}{6} \cdot \frac{\Gamma^{1/2}\,\Gamma 2}{\Gamma 5/2} = \frac{1}{6} \cdot \frac{\Gamma 1/2 \cdot 1}{\frac{3}{2}\cdot\frac{1}{2}\,\Gamma 1/2}$$

$$= \frac{1}{6}\cdot\frac{4}{3} = \frac{2}{9}$$

A

E) $I = \int_0^1 \int_0^x (x+y)\,dy\,dx$

limits are $y=0$ to $y=x$
$x=0$ to $x=1$



convert to polar form

$x = r\cos\theta$

$y = r\sin\theta$

$dxdy = r\,dr\,d\theta$

$r \to 0$ to $1/\cos\theta$

$\theta \to 0$ to $\pi/4$

$x=1$
$r\cos\theta = 1$
$r = 1/\cos\theta$

$$= \int_0^{\pi/4}\int_0^{1/\cos\theta} (r\cos\theta + r\sin\theta)\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/4} (\sin\theta + \cos\theta)\,d\theta \left[\frac{r^3}{3}\right]_0^{\frac{1}{\cos\theta}} \cancel{d\theta}$$

$$= \frac{1}{3}\int_0^{\pi/4} (\sin\theta + \cos\theta)\cdot\frac{1}{\cos^3\theta}\,d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4}\left(\frac{\sin\theta}{\cos^3\theta} + \frac{\cos\theta}{\cos^3\theta}\right)d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4}\left[\tan\theta\cdot\sec^2\theta + \sec^2\theta\right]d\theta$$

$$= \frac{1}{3}\left[\frac{(\tan\theta)^2}{2} + \tan\theta\right]_0^{\pi/4}$$

$$= \frac{1}{3}\left[\frac{1}{2}\cdot 1 + 1\right] = \frac{1}{3}\cdot\frac{3}{2} = \frac{1}{2}$$

F) $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ (15)

$$(D^2-1)y = \frac{2}{1+e^x}$$

AE is $D^2 = 1$

$D = \pm 1$

$y_c = c_1 e^x + c_2 e^{-x}$

Let $y_p = uy_1 + vy_2$

$y_1 = e^x \quad y_2 = e^{-x}$

$y_1' = e^x \quad y_2' = -e^{-x}$

$$w = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -1-1=-2$$

$$u = -\int \frac{y_2 X}{w}\,dx = -\int \frac{e^{-x}}{-2} \cdot \frac{2}{1+e^x}\,dx$$

$$= +\int \frac{e^{-x}}{1+e^x}$$

put $e^{-x} = t \quad -e^{-x}dx = dt$

$$= \int \frac{dt}{1+1/t}\,dt = \int \frac{t}{1+t}\,dt$$

$$= \int \frac{1+t-1}{1+t}\,dt = \int \left(1 - \frac{1}{1+t}\right)dt$$

$$= t - \log(1+t) +$$

$$= e^{-x} - \log(1+e^{-x})$$

$$v = \int \frac{y_1 X}{w}\,dx = \int \frac{e^x}{-2} \cdot \frac{2}{1+e^x}\,dx$$

$$= -\log(1+e^x)$$

$y_p = uy_1 + vy_2$

$y_p = [e^{-x} - \log(1+e^x)]e^x - e^{-x} \cdot \log(1+e^x)$

$y = y_c + y_p$