Duration: 3hrs **[Max Marks:80]**

N.B. : (1) Question No 1 is Compulsory.
(2) Attempt any three questions out of the remaining five.
(3) All questions carry equal marks.
(4) Assume suitable data, statistical tables if required and state it clearly.

1

a Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies Cayley Hamilton theorem [5]

b Prove that **F**= (x+2y+4z)i+(2x-3y-z)j+(4x-y+2z)k is solenoidal [5]

c A discrete random variable has the probability distribution given below [5]

X	0	1	2	3	4	5
P(X=x)	k	3k	5k	7k	9k	11k

Find k, mean

d If A = $\begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$ then show that $A^4 = 5A + 6I$ [5]

2 a Find eigenvalues and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [6]

b Using Green's theorem evaluate $\int (xy + y^2)dx + x^2dy$ where c is the closed curve of the region bounded by y = x and $y = x^2$ [6]

C Investigate the association between the darkness of eye colour in father and son from the following. [8]

Color of son's eyes	Color of father's eyes	
	Dark	Not Dark
Dark	48	90
Not Dark	80	782

3 a If A = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find A^{50} [6]

b By using Big M method solve Minimize $Z = 2x_1 + 3x_2$ Subject to $x_1 + x_2 \geq 5$ [6]

$x_1 + 2x_2 \geq 6$; $x_1\ x_2 \geq 0$

C The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over a week using $\chi 2$ test. [8]

Day	SUN	MON	TUES	WED	THU	FRI	SAT	TOTAL
No. of accidents	13	15	9	11	12	10	14	84

4 a Tests made on breaking strength of 10 pieces of a metal wire gave results 578,572,570,568,572,570,570,572,596,584 kgs. Test if the breaking strength of metal wire can be assumed to be 577 kgs? [6]
(t_{tab} at 5 % LOS =1.833)

b Show that $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalizable. Also find the diagonal form and the transforming matrix [6]

C Solve the following LPP by simplex method [8]

Maximize $Z = 4x_1 + 8x_2 + 5x_3$
Subject to $x_1 + 2x_2 + 3x_3 \leq 18$; $2x_1 + 6x_2 + 4x_3 \leq 15$; $x_1 + 4x_2 + x_3 \leq 6$; $x_1, x_2, x_3 \geq 0$

n

5 a Show that the matrix A is derogatory and find its minimal polynomial A = $\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ [6]

b It is shown that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i)at least 3 (ii) exactly 3 (iii) at most three defective items in a consignment of 1000 packets using Poisson Distribution [6]

C If the vector field $\overline{F}$ is irrotational find the constants a,b,c where $\overline{F}$ = (x + 2y + az)i + (bx – 3y –z)j + (4x + cy + 2z) k. [8]

6 a Use the dual simplex method to solve the following LPP [6]

$$Min.Z = 6x_1 + x_2$$

Subject to the constraints

$$2x_1 + x_2 \geq 3$$

$$x_1 - x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

b A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight. [6]

Diet A : 5 6 8 1 12 4 3 9 6 10 gms

Diet B : 2 3 6 8 1 10 2 8 gms

Find if the variances are significantly different at 5% level of significance.

C Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form and hence find rank, index and signature of the matrix [8]

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