Q.P. Code : 23178

[Time: Three Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. Question.No.1 is compulsory.
2. Attempt any three from the remaining six questions.
3. Figures to the right indicate full marks.

Q.1 a) If the Laplace transform of $\sin^2 3t$ 20
b) Prove that $f\ (z) = \log z$ is analytic
c) Obtain Fourier series for $f\ (x) = x^2$ in (-2,2)
d) Find the Z-Transform of $cos2k, k \geq 0$

Q.2 a) Prove that $\overline{F} = 2xyz^3 i + x^2 z^3 j + 3x^2 yz^2 k$ is irrotational. 06
Find Scalar potential for $\overline{F}$

b) Find the inverse Laplace Transform using Convolution theorem 06
$\frac{1}{(s^2+6s+18)^2}$

c) Find Fourier Series of $f(x) = \frac{\pi - x}{2}$ in $(0,2\pi)$. 08

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + - - - - -$.

Q.3 a) Find the Analytic function $f(z) = u + iv \ \ if\ u + v = \cos x \cosh y - \sin x \sinh y$ 06

b) Find Inverse Z transform of $\frac{2z^2 - 10z + 13}{(z-3)^2 (z-2)}$, $2 < |z| < 3$ 06

c) Solve the Differential Equation $\frac{d^2 y}{dt^2} + 2\ \frac{dy}{dx} y = 3te^{-1}$, $y(0) = 4$, $y'(0) = 2$ using Laplace Transform 08

Q.4 a) Find the Orthogonal Trajectory of $x^2 + y^2 - 3xy + 2y = c$ 06

b) Using Greens theorem evaluate $\int_c (x^2 - y)dx + (2y^2 + x)dy, C$ is closed path formed by $y = 4, y = x^2$ 06

c) Express the function $f\ (x) = \begin{cases} \sin x\ ; 0 < X\ \leq \pi \\ 0 \quad ;\ X > \pi \end{cases}$ as Fourier Integral. Hence evaluate $\int_0^{\infty} \frac{\cos(\lambda\pi/2)}{1-\lambda^2}\, d\lambda$ 08

Q.5

a) Find Inverse Laplace Transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$ 06

b) Find the Bilinear Transformation that maps the points $z = 1, i, -1\ \ into\ w = i, 0, -i$ 06

c) Evaluate using Stoke's theorem $\int_c \bar{F} \bullet \overline{dr}$ where c is the boundary of the circle $x^2 + y^2 + z^2 = 1, z = 0$ and $\bar{F} = yzi + zxj + xyk$ 08

Q.6

a) Find the Directional derivative of $\emptyset = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}\ at\ (1,2,3)$ 06

b) Find complex form of Fourier series for $e^{ax}; (-\pi, \pi)$ 06

c) Find Half Range sine Series for $f(x) = x(2-x) \qquad 0 < x < 2$ hence deduce that $\sum\left(\frac{1}{n^2}\right) = \frac{\pi^6}{945}$ 08

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