

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Figures to right indicate full marks.

- Que. 1**
- Find Laplace Transform of $t \cos 4t \cdot \cos 7t$ 5
 - Find Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$ 5
 - Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ 5
 - If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, Find eigen values of $A^3 - 3A^2 + 5A$ 5
- Que. 2**
- Obtain Fourier series expansion for $f(x) = x^2$ in $(0, 2\pi)$ 6
 - By using partial fractions, find the inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+16)}$ 6
 - Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ 8
- Que. 3**
- Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6
 - Find the Laplace transform of $\sinh^5 t$ 6
 - Using Bender Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to $u(0, t) = 0, u(1, t) = 0, u(x, 0) = \sin \pi x, 0 \leq x \leq 1$ 8
- Que. 4**
- By using Laplace transform, evaluate, $\int_0^\infty \frac{\cos 3t - \cos 5t}{t} dt$ 6
 - Find a, b, c, d, e if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is an analytic function. 6
 - Obtain the half range cosine series of $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$ 8

Que. 5 a. Find the analytic function $f(z) = u + iv$, in terms of z , if $u = y^3 - 3x^2y$ 6

b. If $L\{f(t)\} = \frac{s}{s^2 + s + 4}$, find $L\{e^{-2t} f(2t)\}$ 6

c. Determine if the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ diagonalizable, hence find its diagonal matrix D and modal matrix 8

Que. 6 a. Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2 - x^2)}{12}$, where $0 < x < \pi$. 6

b. Find inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$ 6

c. Using Crank- Nicholson simplified formula, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = \frac{x}{3}(16 - x^2)$ for one step for time. 8
