**(Time: 3 Hours )**                    **Max. Marks: 80**

N.B.    (1) Question No. 1 is compulsory.
        (2) Answer any three questions from Q.2 to Q.6.
        (3) Use of Statistical Tables permitted.
        (4) Figures to the right indicate full marks

Q1.
(a)  Find the Laplace transform of   $\dfrac{\cos 2t \sin t}{e^t}$                                        [ 5 ]

(b)  Find  k such that $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\frac{kx}{y}$   is analytic        [ 5 ]

(c)  Calculate the Spearman's rank correlation coefficient R                                              [ 5 ]
 X   :  10 , 12 , 18 , 18 , 15 , 40 .
 Y   :  12 , 18, 25 , 25 , 50 , 25 .

(d)  Find the inverse Laplace transform of     $\log\left(\dfrac{s^2 + a^2}{s^2 + b^2}\right)$.            [ 5 ]

 Q2.
(a) A continuous random variable has probability density function
            f ( x ) =  k ( $x - x^2$ ) ,      $0 \le x \le 1$.
                              f (x) = 0                    otherwise
        Find  k , mean and variance.                                                                       [ 6 ]

(b) Find the Laplace transform of     $e^{-3t} \int_0^t u \sin 3u \, du$.                                  [ 6 ]

 (c)  Obtain the Fourier series to represent f (x) = $x^2$  in (0, 2$\pi$ )
        Hence show that  $\dfrac{\pi^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2}$ …….            [ 8 ]

Q3.
(a) If the imaginary part of the analytic function w = u + i v = f ( z ) is                               [ 6 ]
        $V = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$ , then show that  u = - 2 x y + $\dfrac{y}{x^2 + y^2}$ .

(b)  Find inverse Laplace transform of  $\dfrac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11 s - 6)}$                  [ 6 ]

(c) Fit a second-degree parabolic curve  and estimate y when  x = 10
     x          : 1 ,   2 ,   3 ,   4 ,   5 ,    6 ,   7 ,   8 ,   9 ,
     y          : 2 ,   6 ,   7 ,   8 ,   10 ,   11 ,   11 ,   10 ,   9 .                                 [ 8 ]

Q4.
 (a)   Obtain the  Fourier series to represent f ( x ) = $x^3$ in ( -$\pi$, $\pi$ ).                      [ 6 ]

 (b)  Find ( i ) the  equation of  the lines of Regression  ( ii ) coefficient of correlation for
      the following data
      X :       65 ,   66 ,   67 ,   67,   68 ,   69 ,   70 ,   72 .
      Y :       67 ,       68 ,   65 ,   66 ,   72 ,   72 ,   69 ,   71 .                                  [ 6 ]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \dfrac{\sin t \sin ht}{t}$ dt = $\dfrac{\pi}{8}$.             [ 8 ]

**37856**                              **Page 1 of 2**

Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$. [ 6]

(b) Find the moment generating function of the distribution

| X | : | -2 | 3 | 1 |
|---|---|----|---|---|
| P X = x ) | : | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{6}$ |

hence find first four central moments . [ 6 ]

(c) Obtain the half range cosine series of $f ( x ) = x$ in $( 0, 2 )$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4}$ …….. [ 8 ]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2^2)^2}\right]$ [ 6 ]

(b) The probability density function of a random variable X is

| X | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|----|----|-------|--------|--------|--------|
| P ( X =x ) : | | k | 2k | 3k | $k^2$ | $k^2$ + k | $2 k^2$ | $4k^2$ |

Find k , p ( X < 5 ), P ( X > 5 ) [ 6 ]

.(c) If $v = 3 x^2 y + 6 x y - y^3$ ,show that v is harmonic function

And find the corresponding analytic function . [ 8 ]

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