Q. P. Code : 22297

Duration – 3 Hours **Total Marks : 80**

(1) N.B.:- Question no 1 is compulsory.

(2) **Attempt any THREE questions out of remaining FIVE questions.**

Q.1) a) Solve $\frac{dy}{dx} = \frac{a^2 - 2xy - y^2}{(x+y)^2}$ (4)

b) *Solve* $(D^3 - 3D^2 + 4)y = 0$ (3)

c) Evaluate $\int_0^\infty e^{-\left(x^2/4\right)} dx$ (3)

d) Express the following integral in polar co-ordinate **(4)**

$$\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} f(x, y)\, dx\, dy$$

e) Prove that $E = 1 + \Delta = e^{hD}$ (3)

f) Evaluate $I = \int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y)\, dy\, dx$ **(3)**

Q.2 a) Solve $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^3$ (6)

b) Change the order of integration and evaluate $I = \int_0^1 \int_{x^2}^{2-x} \frac{x}{y}\, dy\, dx$. (6)

c) Show that $\int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx = \frac{\pi}{2} \log\left(\frac{a}{b}\right)$ **(8)**

Q.3 a) Evaluate $I = \int_0^2 \int_0^y \int_{x-y}^{x+y} (x + y + z)\, dx dy dz$ (6)

b) Find the mass of a plate in the form of a cardiode $r = a(1 - \cos\theta)$ if the density at any point of the plate varies as its distance from the plate. (6)

c) Solve $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = x^2$ **(8)**

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Q. 4 a) Show that the length of the curve $x = a e^{\theta} \sin \theta$. $y = a e^{\theta} \cos \theta$ from $\theta = 0$ to $\theta = \pi/2$ (6)

b) Solve $\frac{d^2 y}{dx^2} - y = \cos x \cosh x + a^x$ (6)

c) Using fourth order Runge-Kutta method, solve numerically, the differential equation $\frac{dy}{dx} = x^2 + y^2$ with the given condition x = 1, y = 1.5 in the interval (1, 1.2) with h = 0.1 **(8)**

Q. 5 a) Use method of variation of parameters to solve $\frac{d^2 y}{dx^2} + y = 3x - 8 \cot x$. (6)

b) Using Taylor's series method, obtain the solution of $\frac{dy}{dx} = y - xy$, $y(0) = 2$. Find the value of y for x = 0.1 correct to four decimal places (6)

c) Evaluate $\int_{-1}^{1} \frac{dx}{1 + x^2}$ by using (i) Trapezoidal Rule, (ii) Simpson's $(1/3)^{rd}$ Rule and (iii) Simpson's $(3/8)^{th}$ Rule. Compare the result with exact solution. **(8)**

Q. 6 a) In a circuit of resistance R, self inductance L, the current i is given by by $L \frac{di}{dt} + Ri = E \cos pt$ where E and p are constants. Find the current i at time 't' (6)

b) Find the area bounded by the parabola $y = 4x - x^2$ and the line $y = x$ (6)

c) Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. **(8)**

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