Q. P. Code: 24392

(3 Hours) [Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q 1. a) Evaluate the Laplace transform of $\sinh(\frac{t}{2})\sin(\frac{\sqrt{3}}{2}t)$ [5]

b) Determine the constants a,b,c,d so that the function $f(z)=x^2+axy+by^2+i(cx^2+dxy+y^2)$ is analytic. [5]

c) Find a unit normal to the surface $xy^3z^2=4$ at the point (-1,-1, 2). [5]

d) Obtain half range sine series for $f(x)=x, \quad 0<x<2$. [5]

Q 2. a) If $u=e^{2x}(x\cos 2y - y\sin 2y)$ then find analytic function $f(z)$ by Milne Thomson Method [6]

b) Find the Fourier series for $f(x)=9-x^2$, $-3\le x\le 3$ [6]

c) Find the Laplace transform of the following

i) $L[t\sqrt{1+\sin t}]$ ii) $L\left[\frac{\sinh 2t}{t}\right]$ [8]

Q 3. a) Prove that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ [6]

b) Evaluate inverse Laplace transform using Convolution Theorem $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ [6]

c) Show that $\overline{F}=ye^{xy}\cos z\,\hat{i}+xe^{xy}\cos z\,\hat{j}-e^{xy}\sin z\,\hat{k}$ is irrotational vector field. Find ϕ if $\overline{F}=\nabla\phi$ and also evaluate $\int_P^Q \overline{F}.d\overline{r}$ along a curve joining the points P(0,0,0) and Q(-1,2, π). [8]

Q 4 a) Find the Fourier transform of $f(t)=e^{-|t|}$ [6]

b) Show that the function $f_1(x)=1$, $f_2(x)=x$ are orthogonal on $(-1,1)$ and determine the constant A & B so that functions $f_3(x)=1+Ax+Bx^2$ is orthogonal to both $f_1(x)$ and $f_2(x)$ on that interval. [6]

c) Find bilinear transformation which maps the points z=1, i,-1 onto the points w=i, 0,-i hence find the image of $|z|<1$ on to w plane find invariant points of this transformation [8]

Q 5 a) solve Using the Laplace transform the following system of equations [6]

$$\frac{dX}{dt} = 2X - 3Y, \frac{dY}{dt} = Y - 2X \quad where \; X(0) = 8, Y(0) = 3.$$

b) Find Complex form of the Fourier series for $f(x) = e^{ax}$ in $-\pi < x < \pi$ where 'a' is a real constant. Hence deduce that $\frac{\pi}{a \sinh a\pi} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^2 + a^2}$ [6]

c) Verify Green's Theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = x^2$ and $y = \sqrt{x}$. [8]

Q 6. a) Prove that $J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ [6]

b) Find the map of the line x-y=1 by transformation $w = \frac{1}{z}$ [6]

c) Evaluate $\iint_S \overline{F} . d\overline{s}$ where $\overline{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ where S is the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ using Gauss divergence theorem. [8]

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