

(Time: 3 hours)

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.**(2) Answer any three questions from Q.2 to Q.6.****(3) Use of Statistical Tables permitted.****(4) Figures to the right indicate full marks.**

Q1. (a) Find all the basic solutions of the following system of equations. Identify the solution which are degenerate and feasible [5]

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

(b) Evaluate $\int_c (z - z^2) dz$ where c is the upper half of the circle $|z| = 1$ [5]

What is the value of the integral for the lower half of the same circle?

(c) Can it be concluded that the average life span of an Indian is more than 70 years, [5]
if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation of 8.9 years.

(d) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then Prove that $3 \tan A = A \tan 3$ [5]

Q2. (a) Evaluate $\int_c \frac{z+6}{z^2-4} dz$ where c is the circle [6]

(i) $|z| = 1$ (ii) $|z - 2| = 1$ (iii) $|z + 2| = 1$

(b) Solve by Simplex Method [6]

$$\text{Max } z = 5x_1 + 7x_2$$

$$\text{S. t } x_1 + x_2 \leq 4$$

$$3x_1 - 8x_2 \leq 24$$

$$10x_1 + 7x_2 \leq 35$$

$$x_1, x_2 \geq 0$$

- (c) In an experiment on immunization of cattle from Tuberculosis , [8]
the following results were obtained.

	Affected	Not Affected	Total
Inoculated	267	27	294
Not inoculated	757	155	912
Total	1024	182	1206

Use χ^2 test to determine the efficacy of vaccine in preventing Tuberculosis.

- Q3. (a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ [6]

- (b) The marks obtained by 1000 students in an examination are found to be [6]

Normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be (i) between 60 & 75 (ii) more than 75

- (c) Obtain Taylor's and Laurent's series expansion of the function [8]

$$f(z) = \frac{2}{(z-1)(z-2)} \text{ when (i) } |z| < 1 \text{ (ii) } 1 < |z| < 2 \text{ (iii) } |z| > 2$$

- Q4. (a) Test the significance of the difference between the means of two normal population the same standard deviation from the following data [6]

	Size	Mean	S.D
Sample 1	100	64	6
Sample 2	200	67	8

- (b) Using Residue theorem evaluate $\int_0^{2\pi} \frac{d\theta}{17-8\cos\theta}$ [6]

- (c) (i) The incidence of an occupational disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random 4 or more will be suffering from the disease. [4]

- (ii) Assume that the probability of an individual coal miner being killed in a mine accident during a year is $1/2400$. Use appropriate statistical distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident every year. [4]

Q5. (a) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. [6]

(b) A medicine was found to be effective for 9 patients in 8 days on an average with standard deviation of 2.2 days. Another medicine administered to another group of 8 patients was found to be effective in 6 days on an average with standard deviation of 2.6 days. Use 5% level of significance to test the null hypothesis that the two medicines are equally effective. [6]

(c) Solve by Dual Simplex Method [8]

$$\text{Min } z = 2x_1 + x_2$$

$$\text{S. t } 3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Q6. (a) Verify Cayley-Hamilton theorem for the matrix [6]

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \text{ and hence find } A^{-1}$$

(b) Use Kuhn Tucker Method to solve the NLPP [6]

$$\text{Max } z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

$$\text{S. t } 3x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

(c) A discrete random variable has the probability density function given below [8]

x	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	k

Find k , mean and Variance.
