Duration: 3hrs   **[Max Marks: 80]**

N.B. : (1) Question No 1 is Compulsory.
      (2) Attempt any three questions out of the remaining five.
      (3) All questions carry equal marks.
      (4) Assume suitable data, if required and state it clearly.

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  a Find $L\left\{\frac{sint}{t}\right\}$ [05]

  b Find the Fourier coefficient of the function $f(x) = x^2$ in $(-\pi, \pi)$ [05]

  c Determine the constants a, b, c, d, e if   [05]

      $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is

      analytic.

  d Given $6y = 5x + 90, 15x = 8y + 130, \sigma_x^2 = 16$.   [05]
      Find i) the means of x and y ii) r iii) σ_y^2

2 a Using convolution theorem find $L^{-1}\left\{\frac{1}{s^2(s+5)^2}\right\}$ [06]

  b Find orthogonal trajectories of $x^3y - xy^3 = c$ [06]

  c Solve using Bender-Schmidt method: $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$; subject to the conditions   [08]
      $u(0, t) = 0; u(4, t) = 0; u(x, 0) = x(4 - x)$ taking $h = 1$ and find the values of u
      upto $t = 5$ seconds.

3 a Find the Complex form of Fourier series for $f(x) = e^x$ in $(0, 2\pi)$ [06]

  b Obtain Taylor's and Laurent's expansions of   [06]
      $f(z) = \frac{z-1}{(z+1)(z-3)}$ in $1 < |z| < 3$

  c Find Laplace transform of $f(t) = t \int_0^t e^{-2u} \sin 4u \, du$ [08]

4 a Production (in metric kiloton) of wheat in a country is given by the following data.   [06]

X	2005	2007	2009	2011	2013	2015	2017
Y	8	12	15	19	21	22	25

      Fit a straight line to the following data and estimate the production in the year 2010.

  b Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ [06]

  c Evaluate $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} \, dz$ where C is the circle $|z| = 1$ [08]

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5 a If $v = 3x^2y + 6xy - y^3$, show that v is harmonic and find the corresponding [06]
       analytic function.

   b Obtain the Half Range Fourier Cosine Series of [06]
      $f(x) = x, 0 \leq x \leq 2$ and deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...$

   c Solve using Crank-Nicolson formula: $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$, $0 < x < 1$, $t > 0$ subject to [08]
      the conditions: $u(x, 0) = 0; u(0, t) = 0; u(1, t) = 200\ t$
      Compute u for one step in t division taking $h = 0.25$

6 a [06]

X	12	17	22	27	32
Y	113	119	117	115	121

   Compute Spearman's rank correlation coefficient (R) from the given data.

   b Obtain the Bilinear transformation that transforms the points [06]
      $z = 2, i, -2$ respectively to the points $w = 1, i, -1$

   c Evaluate $\int_0^{2\pi} \frac{d\theta}{17 - 8\cos\theta}$ [08]

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