

University of Mumbai
Examination First Half 2022

Program: BE Electronics and Telecommunication Engineering

Curriculum Scheme: Rev2016

Examination: SE Semester III

Course Code: ECC301 and Course Name: Applied Mathematics-III

Time: 2 hour 30 minutes

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the questions are compulsory and carry equal marks. 2 marks each
1.	If Laplace transform $L\{f(t)\} = F(s)$ then $L\{e^{-at}f(t)\}$ is
Option A:	$F\left(\frac{s}{a}\right)$
Option B:	$F(s+a)$
Option C:	$F(as)$
Option D:	$F(s-a)$
2.	If the Fourier series of $f(x) = x^2, -\pi < x < \pi$ is $a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ then the value of b_n is
Option A:	$\frac{(-1)^n + 1}{\pi^3 n^3}$
Option B:	$\frac{(-1)^n - 1}{\pi n^2}$
Option C:	$\frac{(-1)^n + 1}{\pi^2 n^2}$
Option D:	0
3.	If $\vec{f} = \text{grad}(xy^2 + 2yz + x)$, then $\text{div } \vec{f}$ is
Option A:	$x + z$
Option B:	0
Option C:	$2x$
Option D:	$2y + z$
4.	The orthogonal trajectories for the family of curves $e^{-x} \cos y = \alpha$ is
Option A:	$e^x \sin y = \beta$

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Option B:	$-e^{-x} \cos y = \beta$
Option C:	$e^{-2x} \sin y = \beta$
Option D:	$e^{-x} \sin y = \beta$
5.	Laplace inverse of $\frac{5s-7}{s^2+4}$ is
Option A:	$5 \cos 2t - \frac{7}{2} \sin 2t$
Option B:	$5 \cos 2t + 7 \sin 2t$
Option C:	$5 \sin 2t - 7 \cos 2t$
Option D:	$5 \sin 2t + \frac{7}{2} \cos 2t$
6.	Applying Green's theorem, the value of $\oint_C \vec{f} \cdot d\vec{r}$ where $\vec{f} = 2xy^3\vec{i} + (x^2 + y^2)\vec{j}$ and C is the rectangle given by $x = \pm 2, y = \pm 1$ is
Option A:	5
Option B:	3
Option C:	1
Option D:	0
7.	Laplace transform of $t^{1/2} e^{-5t}$ is
Option A:	$\frac{\Gamma(1/2)}{(s+5)^{1/2}}$
Option B:	$\frac{\Gamma(3/2)}{(s+5)^{1/2}}$
Option C:	$\frac{\Gamma(3/2)}{(s+5)^{3/2}}$
Option D:	$\frac{\Gamma(1/2)}{(s+5)^{3/2}}$
8.	If the half range sine series of $f(x) = x, 0 \leq x < 2$ is $\sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{2} x$ then the value of b_n is

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Option A:	$\frac{4}{n\pi}$
Option B:	$-\frac{4(-1)^n}{n\pi}$
Option C:	$\frac{4(-1)^n}{n\pi}$
Option D:	$-\frac{4}{n^2\pi}$
9.	Laplace inverse of $\log\left(\frac{s+2}{s-5}\right)$ is
Option A:	$\frac{1}{t}(e^{-2t} - e^{5t})$
Option B:	$\frac{1}{t}(e^{5t} - e^{-2t})$
Option C:	$t(e^{5t} - e^{-2t})$
Option D:	$\frac{1}{t}(e^{-5t} - e^{2t})$
10.	If J_n denotes the Bessel function of order n , then the value of $x J_1'$ is
Option A:	$x J_0 - J_1$
Option B:	$x J_0 - 2 J_1$
Option C:	$-x J_0 + 2 J_1$
Option D:	$2x J_0 - J_1$

Q2.	Solve any Four out of Six.	5 marks each
A	Find the Laplace transform of $\int_0^t t \sinh 3t \, dt$.	
B	Find the Laplace inverse of $\frac{s+1}{(s+4)(s^2+1)}$.	
C	Find the Fourier series expansion of $f(x) = x^2$, $-1 < x < 1$.	

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D	Find the analytic function $u + iv$ where $u = \frac{1}{2} \log(x^2 + y^2)$
E	If the position vector of a point is denoted by $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = \vec{r} $ then show that $\text{div}(\vec{r}^n \vec{r}) = (n+3) r^n$.
F	Prove that $\vec{f} = (4xy + 3x^2z)\mathbf{i} + (2x^2 + 2z)\mathbf{j} + (x^3 + 2y)\mathbf{k}$ is conservative. Determine the work done by $\vec{f}$ on displacing a particle from $(1, 0, i)$ to $(2, 1, 1)$.

Q3.	Solve any Four out of Six.	5 marks each
A	Using Laplace transform, evaluate the integral $\int_0^\infty e^{-t} \frac{\sin 3t + \sin 2t}{t} dt$.	
B	Find the Laplace inverse of $\frac{1}{(s^2 + 9)^2}$ using the convolution property.	
C	Find the Fourier series of $f(x) = x^3$, $-\pi < x < \pi$	
D	Find the analytic function $u + iv$ if $u - v = e^x (\cos y - \sin y)$	
E	If $\phi = xyz$ and $\psi = xy + yz + xz$, then find $\nabla \cdot (\nabla \phi \times \nabla \psi)$	
F	Using Gauss divergence theorem, evaluate $\iiint_S 4x\mathbf{i} + 3y\mathbf{j} - 2z\mathbf{k} \cdot d\vec{s}$ where S is the closed surface bounded by $x =$, $y = 0$, $z = 0$, $2x + 2y + z = 4$.	

Q4.	Solve any Four out of Six.	5 marks each
A	Solve the equation $y'' + 9y = t$; $y(0) = -1$, $y'(0) = 0$	
B	Find the Laplace transform of $e^{-t}(1+t+t^2)H(t-3)$	
C	Find the complex form of Fourier series of e^{2x} on $(-\pi, \pi)$	
D	For a scalar function $\Phi = 2xy - y^2 + z^2$, find the directional derivative of	

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	Φ at the point $(1, 0, 1)$ in the direction of vector $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
E	Show that $\int J_3(x) dx = J_0(x) - \frac{4}{x} J_1(x)$
F	Find the Bilinear transformation which maps $z_1 = 1, z_2 = i, z_3 = -1$ onto $w_1 = 0, w_2 = 1, w_3 = \infty$.