

Time Duration: 3Hr

Total Marks: 80

N.B.: 1) Question no.1 is compulsory.

2) Attempt any three questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

- Q1. a) Find the Laplace transform of $e^{-4t}t \sin 3t$. [5]
 b) Find the half-range cosine series for $f(x) = x$, $0 < x < 2$. [5]
 c) Find $\nabla \cdot \left(r \nabla \frac{1}{r^3} \right)$. [5]
 d) Show that the function $f(z) = \sin z$ is analytic and find $f'(z)$ in terms of z . [5]
- Q2. a) Find the inverse Z-transform of $F(z) = \frac{1}{(z-5)^3}$, $|z| < 5$. [6]
 b) Find the analytic function whose imaginary part is $e^{-x}(y \sin y + x \cos y)$. [6]
 c) Obtain Fourier series for the function $f(x) = x + x^2$, $-\pi \leq x \leq \pi$ and $f(x + 2\pi) = f(x)$. [8]
 Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ and $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- Q3. a) Find $L^{-1} \left[\frac{1}{(s-a)(s-b)} \right]$ using convolution theorem. [6]
 b) Is $S = \left\{ \sin \left(\frac{\pi x}{4} \right), \sin \left(\frac{3\pi x}{4} \right), \sin \left(\frac{5\pi x}{4} \right), \dots \right\}$ orthogonal in $(0, 1)$? [6]
 c) Using Green's theorem in the plane evaluate $\int_C (xy + y^2)dx + (x^2)dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [8]
- Q4. a) Find Laplace transform of $f(t) = \begin{cases} \sin 2t & , 0 < t \leq \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < t < \pi \end{cases}$ and $f(t) = f(t + \pi)$. [6]
 b) Prove that a vector field \vec{f} is irrotational and hence find its scalar potential $\vec{f} = (x^2 + xy^2)i + (y^2 + x^2y)j$. [6]
 c) Find the Fourier expansion for $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$. [8]
- Q5. a) Use Gauss's Divergence Theorem to show that $\iint_S \nabla \cdot r^2 \vec{ds} = 6V$ where S is any closed surface enclosing a volume V . [6]
 b) Find the Z-transform of $f(k) = b^k$, $k < 0$. [6]
 c) i) Find $L^{-1} \left[\frac{s}{(s-2)^6} \right]$. [8]
 ii) Find $L^{-1} \left[\log \left(1 + \frac{a^2}{s^2} \right) \right]$.
- Q6. a) Solve using Laplace transform [6]
 $(D^2 + 9)y = 18t$, given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$.
 b) Find the bilinear transformation which maps the points $Z = \infty, i, 0$ onto $W = 0, i, \infty$. [6]
 c) Find Fourier integral representation of $f(x) = e^{-|x|}$, $-\infty < x < \infty$. [8]