3 Hours **Total Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Figures to the right indicate full marks.

Q1.

a) Find the eigen values of $A^2 - 5A + 4I$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ **5**

b) Find the Fourier expansion of $f(x) = x^2, -\pi \le x \le \pi$ **5**

c) Find a , b , c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. **5**

d) Find $L[te^{3t}sint]$ **5**

Q2.

a) Evaluate the following Integral using Laplace Transforms. **6**

$$I = \int_0^\infty \frac{sin^2 t\, e^{-t}}{t} dt$$

b) Determine the Fourier Series $f(x) = \left(\frac{\pi - x}{2}\right)^2$ over $[0, 2\pi]$. **6**

c) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic and find its harmonic conjugate. **8**

Q3.

a) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method subjected to the conditions $u(0,t) = 0$, $u(x,0) = 0, u(1,t) = t$, taking h =0.25, $0 < x < 1$, upto = 5 . **6**

b) Determine the analytic function $f(z) = u + iv$ where $u = 3x^2y - y^3$. **6**

c) Determine the Inverse Laplace Transform of i) $\frac{s+2}{s^2 - 4s + 13}$ **4**

ii) $tan^{-1}(s)$ **4**

Q4.

a) i) If $L\{f(t)\} = \frac{s}{s^2+s+4}$, find $L\{e^{-2t}\ f(2t)\}$ **3**

ii) Find $L(t^2\ sinat)$ **3**

b) Determine the Inverse Laplace Transform of $\log\left[\frac{s^2+a^2}{(s+b)^2}\right]$ **6**

c) $Is\ the\ matrix\ A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ diagonalizable? If so find the diagonal form of A and transforming matrix of A. **8**

Q5.

a) Find the Eigen value and the eigen vector of A= $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$ **6**

b) Find Inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$ using partial fraction method. **6**

c) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, by Crank-Nicholson simplified formula, where $u(0,t) = 0, u(4,t) = 0, u(x,0) = \frac{x}{3}(16 - x^2)$, find u_{ij} , for $i = 0,1,2,3,4\ and\ j = 0,1,2$ taking $h = 1$. **8**

Q6.

a) Find analytic function f(z) whose imaginary part is $e^x\ cosy + x^3 - 3xy^2$ **6**

b) Find the Laplace Transform of $f(t) = \frac{\cos at - \cos bt}{t}$ **6**

c) Determine the Fourier Series for $f(x) = \begin{cases} x + \frac{\pi}{2}, & -\pi \le x \le 0 \\ \frac{\pi}{2} - x, & 0 \le x \le \pi \end{cases}$ over $[-\pi, \pi]$