Duration: 3 hours **Max. Marks: 80**

N.B. (1) Question No. 1 is **COMPULSORY**.
(2) Answer **ANY THREE** questions from Q.2 to Q.6.
(3) Figures to right indicate full marks.

Que. 1	**a.**	Find Laplace Transform of $tcos4t.cos7t$	**5**
	b.	Find Fourier series expansion of $f(x) = x \ \ in \ (-\pi, \pi)$	**5**
	c.	Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$	**5**
	d.	If $A=\begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, Find eigen values of A^3-3A^2+5A	**5**
Que. 2	**a.**	Obtain Fourier series expansion for f (x) = x^2 in $(0, 2\pi)$	**6**
	b.	By using partial fractions, find the inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+16)}$	**6**
	c.	Find the eigenvalues and the eigenvectors of the matrix $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$	**8**
Que. 3	**a.**	Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$	**6**
	b.	Find the Laplace transform of $sinh^5 t$	**6**
	c.	Using Bender Schmidt method , solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to $u(0,t) = 0, \ u(1,t) = 0, \ u(x,0) = \sin \pi x \quad 0 \le x \le 1$	**8**
Que. 4	**a.**	By using Laplace transform, evaluate, $\int_0^\infty \frac{cos3t - cos5t}{t} dt$	**6**
	b.	Find a , b ,c , d, e if $f(z) = (ax^4 + bx^2y^2 + cy^4 + + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is an analytic function.	**6**
	c.	Obtain the half range cosine series of $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$	**8**

Que. 5

a. Find the analytic function $f(z) = u + iv$, in terms of z, if $u=y^3-3x^2y$ **6**

b. If $L\{f(t)\} = \frac{s}{s^2+s+4}$, find $L\{e^{-2t} f(2t)\}$ **6**

c. Determine if the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ diagonalizable, hence find it's diagonal matrix D and modal matrix **8**

Que. 6

a. Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2-x^2)}{12}$, where $0 < x < \pi$. **6**

b. Find inverse Laplace transform of $cot^{-1}\left(\frac{s+3}{2}\right)$ **6**

c. Using Crank- Nicholson simplified formula, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, $u(0,t) = 0,\ u(4,t) = 0,\ u(x,0) = \frac{x}{3}(16-x^2)$ for one step for time. **8**

****