Paper / Subject Code: 41201 / Applied Mathematics-IV Date - 4/12/19

S.E. (Mechanical) (Sem-IV) (CBCGS)

[Time: 3 Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. **Question no. 1** is **compulsory**.
2. Answer **any three** from **remaining**.
3. **Figures** to the right **indicate** full **marks**.
4. **Use** of statistical **tables** is **allowed**.

Q.1 a) Find eigen values of A^3-2A^2+I and adj A **(05)**

where $A = \begin{bmatrix} 4 & 1 & -1 \\ 6 & 3 & -5 \\ 6 & 2 & -2 \end{bmatrix}$

Q.1 b) A random variable X has the following probability function. **(05)**

X	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	4K	6K	4K	K

Find (i) K (ii) $P(X<4)$ (iii) $P(X>3)$ (iv) $P(0<X\le 2)$

Q.1 c) Can it be concluded that the average life-span of an Indian is more than 71 years, if a random sample of 900 Indians has an average life span 72.8 years with standard deviation of 7.2 years? **(05)**

Q.1 d) Consider the following problem: **(05)**

Maximize $Z = 2x_1-2x_2+4x_3-5x_4$

Subject to $x_1 + 4x_2 - 2x_3 + 8x_4 = 2,$

$-x_1 + 2x_2 + 3x_3 + 4x_4 = 1,$

$x_1, x_2, x_3, x_4 \ge 0$

Find a basic feasible solution which is non-degerate and optimal solution.

Q.2 a) Check whether the given matrix A is diagonalizable, diagonalize if it is, **(06)**

Where $A = \begin{bmatrix} 8 & 4 & 3 \\ -8 & -3 & -4 \\ -2 & -2 & 1 \end{bmatrix}$

Q.2 b) Verify Green's theorem for $\vec{F} = x^2 i - xyj$ where C is the triangle having vertices A(0,3), B(3,0), C(6,3). **(06)**

Q.2 c) Sample of two types of electric bulbs were tested for length of life and the following data were obtained, **(08)**

	Type I	Type II
Sample size	10	9
Mean of the sample (in hours)	1136	1034
Standard deviation (in hours)	36	39

Test at 5% level of significance whether the difference in the sample means is significant.

Q.3 a) Use the dual simplex method to solve the following LPP. **(06)**

Minimise $Z = 6x_1 - x_2$

Subject to $2x_1 + x_2 \geq 3$,

$x_1 - x_2 \geq 0$,

$x_1, x_2 \geq 0$.

Q.3 b) Use Gauss Divergence Theorem to evaluate $\iint_S \bar{N}.\bar{F}\, ds$ where $\bar{F} = 2xi + 2yj + 2z^2k$ **(06)** and S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

Q.3 c) Find the rank, index, signature and class of the following Quadratic form by reducing it to its canonical form. **(08)**

$2x^2-2y^2+2z^2-2xy-2yz+6zx$.

Q.4 a) Four dice were thrown 250 times and the number of appearance of 6 each time was noted. **(06)**

No. of successes (x):	0	1	2	3	4
Frequency (f):	133	69	34	11	3

Fit a poisson distribution and find the expected frequencies for $x = 0, 1, 2, 3, 4$.

Q.4 b) Verify Cayley Hamilton theorem for matrix A and hence find the matrix represented by **(06)**

$A^5-4A^4-7A^3+11A^2-A-11I$

Where $A = \begin{bmatrix} 3 & -2 & 3 \\ 10 & -3 & 5 \\ 5 & -4 & 7 \end{bmatrix}$

Q.4 c) An investigation into the equality of standard deviation of two normal populations gave the following results. **(08)**

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	13	18	105
2	21	24	145

Examine the equality of sample variances at 5% level of significance.
(Given: $F_{0.025} = 2.68$ for d.o.f 12 and 20 and $F_{0.025} = 3.07$ for d.o.f 20 and 12)

Q.5 a) Is matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 3 & -1 \\ 3 & -1 & 3 \end{bmatrix}$ Derogatory matrix? Find its minimal polynomial. **(06)**

Q.5 b) A vector field $\bar{F}$ is given by **(06)**
$\bar{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)i + (xy\cos z + y^2)k$
Prove that $\bar{F}$ is irrotational. Hence find its scalar potential function φ if $\varphi(\pi, l, o)$.

Q.5 c) The following table gives the result of opinion pole for three vehicles A, B, C. Test whether the age and the choice of the vehicle are independent at 5% level of significance using χ^2 – test. **(08)**

Age	Vehicle			Total
	A	B	C	
20-35	25	40	35	100
35-50	35	24	41	100
Above 50	40	36	24	100
Total	100	100	100	300

Q.6 a) State stoke's theorem and evaluate $\int[(x^2 + y^2)i + (x^2 - y^2)j].d\bar{r}$ **(06)**
Where C is the square in the xy-plane with vertices (1,0), (0,1), (-1,0) and (0,-1)

Q.6 b) Monthly salary X in an organisation is normally distributed with mean Rs. 3000 and standard deviation of Rs. 250. What should be the normally minimum salary of an employee in this organisation so that the probability that an employee to top 5% employees? **(06)**

Q.6 c) Using duality solve the following LPP, **(08)**

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 5$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0.$

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