[3 Hours] **[Total Marks: 80]**

Note :
1) Q. No. 01 is compulsory.
2) Solve any three from Q. No. 02 to 06.
3) Numbers to the right indicates full marks

Q. 1. Solve.

a) If $f(t) = t, 0 < t < 4,\ f(t) = 5, t > 4$. Find the L. T. of$f(t)$. 5

b) Find the Z – transform of $\left(\frac{1}{2}\right)^k, k \geq 0$. Also give the region of convergence. 5

c) Find the constants k if ,

$f(z) = \frac{1}{2}\log(x^2 + y^2) + itan^{-1}\left[\frac{kx}{y}\right]$ is analytic. 5

d) Fit a straight to the data 5

X :	1	2	3	4	5	6
Y :	49	54	60	73	80	86

Q. 2.

a) Solve $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 20\ Sin2t$, given $y(0) = 1, y'(0) = 2$. 6

b) Find the Bilinear transformation which maps the points 0, 1, ∞ of the Z-plane to $-5, -1, 3$ of the W- plane. 6

c) Find the Fourier expansion of $f(z) = 1 - x^2$, in (- 1, 1). 8

Q. 3.

a) Find Correlation coefficient of the data 6

X:	10	12	18	18	15	40;
Y:	12	18	25	25	50	25.

b) Find inverse Z – transform of $\frac{1}{(z-1)(z-3)}$ for the region $|z| < 1, |z| > 3$. 6

c) Obtain Fourier Series for the function $f(x) = 1 + \frac{2x}{\pi}$, $-\pi \leq x \leq 0$
$= 1 - \frac{2x}{\pi}$, $0 \leq x \leq \pi$ 8

Q. 4.

a) Obtain half range sine series for $f(x) = x$, in $(0, \pi)$. 6

b) Find the orthogonal trajectories of family of the curve $2x - x^3 + 3xy^2 = k$. 6

c) Find the Laplace Transform of i) $e^{-2t}tSin\ 3t$, ii) $\frac{Cos\ 4t - Cos\ 6t}{t}$. 8

Q. 5.

a) Find inverse L. T. by using Convolution theorem. $\frac{2s}{(s^2+4)^2}$. 6

b) Given 6y = 5x + 90, 15x = 8y + 130 & Var(x) = 16. Find means of x, y, Var (y) & r. 6

c) Show that the functions $f_1(x) = 1, f_2(x) = x$ are orthogonal on $(-1, 1)$. Determine the constants a, b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that interval. 8

Q. 6.

a) Find an analytic function whose imaginary part is $e^{-x}(ysiny + xcosy)$. 6

b) Evaluate $\int_0^\infty e^{-2t}tsin^2tdt$. 6

c) Find inverse Laplace Transform of i) $\frac{s}{(3s+1)^2}$ ii) $\frac{s+2}{s^2+4s+9}$. 8
