Time: 3 Hours **Total Marks: 80**

Instructions:

1) **Question 1 is compulsory**
2) **Attempt any three from the remaining questions.**

1-a) If $y = 2^x \operatorname{Sin}^2 x \operatorname{Cos} x$ find y_n. (5 Marks)

1-b) State Euler's theorem on homogeneous function of two variables and evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where, $u = \frac{x+y}{x^2+y^2}$. (5 Marks)

1-c) Separate into real and imaginary part of $\cos^{-1}\left(\frac{3i}{4}\right)$. (5 Marks)

1-d) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. (5 Marks)

2-a) If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ P.T $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$ (6 Marks)

2-b) Show that $\frac{\operatorname{Sin}5\theta}{\operatorname{Sin}\theta} = 16\operatorname{Cos}^4\theta - 12\operatorname{Cos}^2\theta + 1$ (6 Marks)

2-c) Test for consistency the following system & solve them if consistent. (8 Marks)

$$x_1 - 2x_2 + x_3 - x_4 = 2$$
$$x_1 + 2x_2 + 2x_4 = 1$$
$$4x_2 - x_3 + 3x_4 = -1$$

3-a) Show that minimum value of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$. (6 Marks)

3-b) Using Newton-Raphson method find the root of equation $2x^3 - 3x + 4 = 0$ lying between -2 and -1 correct to four places of decimals. (6 Marks)

3-c) If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

(8 Marks)

4-a) Apply Gauss elimination method to solve the equations x+3y-2z=5, 2x+y-3z=1, 3x+2y-z=6. (6 Marks)

4-b) Solve $x^5 = 1 + i$ and find the continued product of the roots. (6 Marks)

4-c) For what value of λ the equations $x + 2y + z = 3, x + y + z = \lambda, 3x + y + 3z = \lambda^2$ have a solution and solve them completely in each case. (8 Marks)

5-a) Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$.

(6 Marks)

5-b) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$. (6 Marks)

5-c) Prove that $\log\left[\frac{sinx + iy}{sinx - iy}\right] = 2i \tan^{-1}(cotx\ tanhy)$ (8 Marks)

6-a) Find the n^{th} derivative of $\frac{x}{(x-1)(x-2)(x-3)}$ (6 Marks)

6-b) Reduce the following matrix to its normal form and hence find its rank.

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$ (6 Marks)

6-c) i) Express $(2x^3 + 3x^2 - 8x + 7)$ in terms of $(x - 2)$ using Taylor's theorem.

ii) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots$ (8 Marks)

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