 Date - 4/12/19

S.E. (Mechanical) (Sem-IV) (CBSGS) (R-2012)

Duration: 3 hours **Total Marks: 80**

N.B **(1) Q.1 is compulsory.**
(2) Solve any three out of the remaining from Q.2 to Q.6.
(3)Figure to right indicate full marks
(4)Use of statistical table is allowed.

Q.1.a. If λ is an Eigen value of a non-singular matrix A, prove that $\frac{|A|}{\lambda}$ is an Eigen value of adj.A **(5)**

b In a Poisson distribution, P(x=3) is 2/3 of P(x=4). Find the mean and the standard deviation **(5)**

c Find the angle between the normals to the surface $x\,y = z^2$, at the points (1 ,4 ,2) & (-3 ,-3 ,3) **(5)**

d A random sample of 50 items given the mean 6.2 and variance 10.24. Can it be regarded as drawn from a Normal population with mean 5.4? **(5)**

Q.2.a A vector field is given by $\bar{F} = (x^2 + xy^2)\,i + (y^2 + x^2 y)\,j$, prove that it is irrotational and hence , find its scalar potential **(6)**

b Following results were obtained from 2 samples each drawn from the different population A & B. **(6)**

Population	A	B
Sample size	21	17
Sample SD	45	40

Test the hypothesis that variance of A is less than or equal to variance of B

c Show that the matrix A = $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and the diagonalising marix M **(8)**

Q.3.a In a large institution 2.28% employees receive income below Rs.4500 and 15.87% employees receive income above Rs.7500. Assuming the income to be normally distributed, find the mean and standard deviation? **(6)**

b Using the method of Lagrange's multipliers , solve the following NLPP **(6)**
Optimise $Z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$
Subject to $x_1 + x_2 + x_3 = 10$
$X_1, x_2, x_3 \geq 0$

c Evaluate $\int_c \bar{F}.d\bar{r}$ where $\bar{F} = \text{Cos } y\,.i - x \text{ Sin } y\,.j$ and c is the curve $y = \sqrt{1-x^2}$ in the xy plane from (1,0) to (0,1) **(8)**

Q.4.a The height of 6 randomly chosen sailors are in inches: 63, 65 , 68, 69, 71, 72. The heights of 10 randomly chosen soldiers are: 61, 62, 65, 66, 69, 69, 70, 71, 72, 73. Discuss in the light that these data throw on the suggestion that the soldiers on an average are taller than the sailors. **(6)**

b Obtain the rank correlation from the following data **(6)**

X	40	42	45	35	36	39
Y	46	43	44	39	40	43

c If $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$, find cos A **(8)**

Q.5.a Out of 1000 families with 4 children each , how many would you expect to have **(6)**
(1) 2 boys & 2 girls (2) at least one boy (3) no girls (4) at most 2 girls

b In an experiment on immunization of cattle from TB, the following results were obtained. **(6)**

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use Chi-square test to determine the efficacy of vaccine in preventing TB

c Using Kuhn-Tucker conditions , solve the following NLPP **(8)**
Maximize $Z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$
Subject to $2x_1 + x_2 \le 5$
$x_1, x_2 \ge 0$

Q.6.a Using Green's theorem, evaluate $\oint_c (e^{x^2} - xy)\, dx - (y^2 - ax)dy$ where c is the circle $x^2 + y^2 = a^2$ **(6)**

b Suppose that in a certain region , the daily rainfall (in inches) is a continuous random variable X with probability density function f(x) is given by **(6)**

$$f(x) = \frac{3}{4}(2x - x^2),\ 0 \le x \le 2$$
$$= 0 \text{, elsewhere}$$

Find the probability that on a given day in this region, the rain fall is (1) not more than 1 inch (2) greater than 1.5 inches (3) between 0.5 and 1.5 inches

c Find the coefficient of regression and hence the equations of the lines of regression for the following data. **(8)**

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

Estimate the value of y when x = 50 and also estimate the value of x when y = 90

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