Q. P. Code: 21237

Time: 3 Hours **Marks: 80**

Note : 1) Q.1 is COMPULSORY.

2) Attempt ANY 3 questions from Q.2 to Q.6

3) Use of scientific calculators allowed.

4) Figures to right indicate marks.

Q.1 a) Find the Laplace transform of $t\,e^{t}\sin 2t\cos t$. (05)

b) Find the inverse Laplace transform of $\frac{s+2}{s^2(s+3)}$ (05)

c) Determine whether the function $f(z) = x^2 - y^2 + 2ixy$ is analytic and if so find its derivative. (05)

d) Find the Fourier series for $f(x) = e^{-|x|}$ in the interval $(-\pi, \pi)$. (05)

Q.2 a) Evaluate $\int_0^{\infty} \frac{e^{-t} - \cos t}{te^{4t}}\,dt$ (06)

b) Find the Z- Transform of $f(k) = \begin{cases} 3^k, & k < 0 \\ 2^k, & k \geq 0 \end{cases}$ (06)

c) Show that the function $u = 2x(1 - y)$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function. (08)

Q.3 a) Find the equation of the line of regression of y on x for the following data (06)

X	10	12	13	16	17	20	25
y	19	22	24	27	29	33	37

b) Find the bilinear transformation which maps $z = 2, 1, 0$ onto $w = 1, 0, i$. (06)

c) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half range cosine series.

Hence show that $\sum_1^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$. (08)

Q.4 a) Find the inverse Laplace Transform by using convolution theorem

$\frac{1}{(s^2+1)(s^2+9)}$ (06)

b) Calculate the coefficient of correlation between Price and Demand. (06)

Price : 2, 3, 4, 7, 4.

Demand : 8, 7, 3, 1, 1.

c) Find the inverse Z-transform for the following ; (08)

i) $\frac{z}{z-5}$, $|z| < 5$ ii) $\frac{1}{(z-1)^2}$, $|z| > 1$

Q.5 a) Find the Laplace transform of e^{-t} sint $H(t - \pi)$ (06)

b) Show that the set of functions { sinx , sin3x , sin5x , …….} is orthogonal over [0, π/2]. Hence construct orthonormal set of functions. (06)

c) Solve using Laplace transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3t\,e^{-t}$, given y(0) = 4 and y ’(0) = 2. (08)

Q.6 a) Find the complex form of Fourier series for f(x) = 3x in (0, 2π). (06)

b) If f(z) is an analytic function with constant modulus then , prove that f(z) is constant. (06)

c) Fit a curve of the form $y = ax^b$ to the following data. (08)

x	1	2	3	4
y	2.5	8	19	50

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