S.E.(Mechanical) (Sem-III) (CBSGS) Date-14/11/2019

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right) \cos\theta$. 5

b) Find Laplace transform of $f(t) = e^{-3t} \sin h2t . \cos 3t$. 5

c) Obtain Fourier series for $f(x) = x$ in $(-\pi, \pi)$. 5

d) Evaluate $\int_C |z|\, dz$ where C is the left half of unit circle $|z| = 1$ from $z = -i$ to $z = i$. 5

2. a) Obtain the Taylor's and Laurent series which represent the function $f(z) = \frac{z+2}{(z-1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

b) Obtain Complex form of Fourier series for $f(x) = \sin hx$ in $(-\pi, \pi)$ 6

c) Using Laplace transform, solve the differential equation:

$\frac{dx}{dt} + 2x = \sin\omega t$, with $x(0) = 1$. 8

3. a) Using Bender –Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions,

$u(0,t) = 0, u(4,t) = 0, u(x,0) = x^2(16 - x^2)$ taking $h = 1$, for 3 minutes. 6

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 6

c) Obtain half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x \le \pi/2 \\ \pi - x, & \pi/2 \le x < \pi \end{cases}$ 8

Hence, prove that –

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

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SE. (Mechanical) (Sem-III) (CBSGS)

4. a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = c$ 6

b) Using Convolution theorem, find the inverse Laplace transform of, 6

$$F(s) = \frac{1}{s(s^2 + 16)}$$

c) Find the Fourier series for $f(x) = x|x|$ in $(-1, 1)$. 8

5. a) Solve by Crank -Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, 6

$u(0,t) = 0,\ u(4,t) = 0, u(x,0) = \frac{x}{3}(16 - x^2)$taking $h = 1$ for 2-time steps.

b) Find the image of the circle $|z| = 4$ in the z-plane under the transformation $w = z + 3i$. Draw the sketch. 6

c) Find the analytic function $f(z) = u + iv$ if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$$

when $f\left(\frac{\pi}{2}\right) = 0$. 8

6. a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$ 6

b) Using Laplace transform, evaluate $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$. 6

c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0, u(l,t) = 0, u(x,0) = x, (0 < x < l), l$ being the length of the rod. 8

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