Time: 3 hour Max. Marks: 80

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Note: (1) Question number **1** is **compulsory.**
(2) Attempt any **three** questions from the remaining **five** questions.
(3) **Figures** to the **right** indicate **full** marks.

Q.1

a) State the multiplication by t property and compute the Laplace transform of $t\,e^{-4t}\ \sin 3t$. 05

b) Compute the inverse Laplace transform of $\frac{s+2}{s^2-4s+13}$ 05

c) Obtain the Fourier series for $f(x)=x^2$ in $(-\pi,\pi)$. 05

d) If the imaginary part $v=3x^2y+6xy-y^3$, find the corresponding analytic function. 05

Q.2

a) Show that $\int_0^\infty \frac{\sin 2t+\sin 3t}{t\,e^t}\,dt=\frac{3\pi}{4}$ 06

b) Compute the inverse Laplace transform of $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$. 06

c) Obtain half range sine series for $f(x)=x\sin x$ in $(0,2\pi)$ and hence deduce that $\sum_{n=2}^{\infty}\frac{1}{n^2-1}=\frac{3}{4}$ 08

Q.3

a) Evaluate $\int_0^\infty e^{-t}\left(\frac{\cos 3t-\cos 2t}{t}\right)dt$ 06

b) Find the constants a, b, c, d if $f(z)=x^2+2axy+by^2+i(cx^2+2dxy+y^2)$ is analytic. 06

c) Evaluate by using Green's theorem $\int_C e^{-x}\sin y\,dx+e^{-x}\cos y\,dy$, where C is the rectangle whose vertices are $(0,0)$, $(\pi,0)$, $(\pi,\frac{\pi}{2})$, $(0,\frac{\pi}{2})$ 08

Q.4

a) Find the directional derivative of $\phi=x^4+y^4+z^4$ at A (1, -2, 1) in the direction of line AB where $B\equiv(2,6,-1)$. 06

b) Find $L^{-1}\left[\tan^{-1}\left(\frac{2}{s}\right)\right]$ 06

c) Obtain half range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. Hence find the values of $\sum \frac{(-1)^n}{(2n-1)^3}$ 08

Q.5

a) Find the analytic function whose real part is $e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$. 06

b) Show that $\bar{F}$ is both solenoidal and irrotational where, $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ 06

c) Using convolution theorem find inverse Laplace transform of $\frac{(s+2)^2}{(s^2 + 4s + 8)^2}$ 08

Q.6

a) State True or False with proper justification " There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$ ". 06

b) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$ 06

c) Find the angle between the surfaces $x \log z + 1 - y^2 = 0$, $x^2 y + z = 2$ at $(1,1,1)$ 08

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