

Duration – 3 Hours

Total Marks : 80

N.B.:- 1. Question no 1 is compulsory.**2. Attempt any THREE questions out of remaining FIVE questions.**

Q.1 a) Write the dual of the given LPP (5)

Maximize $Z = 4x_1 + 9x_2 + 2x_3$ Subject to: $2x_1 + 3x_2 + 2x_3 \leq 7$, $3x_1 - 2x_2 + 4x_3 = 5$, $x_1, x_2, x_3 \geq 0$.

b) If X is a Random Variable with probability density function (5)

$$f(x) = \begin{cases} kx; 0 \leq x \leq 2 \\ 2k; 2 \leq x \leq 4 \\ 6k - kx; 4 \leq x \leq 6 \end{cases}$$

Find k, expectation and $P(1 \leq x \leq 3)$.

c) A tyre company claims that the life of the tyres have mean 42,000 kms with standard deviation of 4,000 kms. A change in the production process is believed to a result in better product. A test sample of 81 new tyres has a mean life 42,500 kms. Test at 5% level of significance that the new product is significantly better than the old one. (5)

d) Find the minimal polynomial of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. Is A derogatory? (5)

Q.2 a) Use Big-M method to solve the following LPP (6)

Minimize $Z = 2x_1 + x_2$ subject to $3x_1 + x_2 = 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \leq 3$, $x_1, x_2 \geq 0$

b) Find e^A and 4^A if $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$. (6)

c) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the closed curve given by $y = x^2$, $y = \sqrt{x}$. (8)

Q.3 a) Prove that $\vec{F} = 2xyz^2\vec{i} + (x^2z^2 + z \cos yz)\vec{j} + (2x^2yz + y \cos yz)\vec{k}$ is a conservative field. Find ϕ such that $\vec{F} = \nabla\phi$. Hence find the work done in moving an object in this field from $(0,0,1)$ to $(1, \frac{\pi}{4}, 2)$. (6)

b) The standard deviations calculated from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regarded as drawn from the normal populations with same standard Deviations. (6)

(Given: $F(0.025) = 3.51$ with d. f. 8 & 12 and $F(0.025) = 4.20$ with d. f. 12 & 8.)

- c) Find the index, rank, signature and class of the Quadratic Form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ by reducing it to canonical form using congruent transformation method. (8)
- Q. 4 a) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (2xy + z)i + y^2j - (x + 3y)k$ and S is the closed surface bounded by $x = 0, y = 0, z = 0, 2x + 2y + z = 6$. (6)
- b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and hence find $2A^4 - 5A^3 - 7A + 6I$. (6)
- c) A sample of 400 students of under-graduate and 400 students of post-graduate classes was taken to know their opinion about autonomous colleges. 290 of the under-graduate and 310 of the post-graduate students favoured the autonomous status. Use chi-square test and test that the opinion regarding autonomous status of colleges is independent of the level of classes of students. (8)
- Q. 5 a) Prove that $\nabla \times \left[\frac{\vec{a} \times \vec{r}}{r^3} \right] = \frac{-\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})\vec{r}}{r^5}$ (6)
- b) Show that the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable and hence find the transforming matrix and diagonal matrix. (6)
- c) Ten school boys were given a test in statistics and their scores were recorded. They were given a month special coaching and a second test was given to them in the same subject at the end of the coaching period. Test at 5% level of significance, if the marks given below give evidence to the fact that the students are benefited by coaching. (8)
- Mark in test 1: 70 68 56 75 80 90 68 75 56 58
Mark in test 2: 68 70 52 73 75 78 80 92 54 55
- Q. 6 a) In a sample of 1000 cases, the mean of a certain test is 14 and Standard Deviation is 2.5. Assuming the distribution to be normal, find (6)
- 1] how many students score between 12 & 15.
2] how many score above 18.
- b) Evaluate by Stoke's theorem $\int_C xy \, dx + xy^2 \, dy$, where C is the square in the xy -plane with vertices $(1, 0), (0, 1), (-1, 0), (0, -1)$. (6)
- c) Using duality solve the following L.P.P. (8)
- Minimise $z = 0.7x_1 + 0.5x_2$
subject to $x_1 \geq 4, x_2 \geq 6, x_1 + 2x_2 \geq 20, 2x_1 + x_2 \geq 18,$
 $x_1, x_2, x_3 \geq 0.$